"Oh my goodness. The whole thing is about ratios!"

A Longitudinal Summative Evaluation of Visitor Experiences in Four *Math Moves!* Exhibitions

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May 2016

Suggested citation:

Selinda Research Associates. (2016). *"Oh my goodness. The whole thing is about ratios!" A longitudinal summative evaluation of visitor experiences in four Math Moves! exhibitions*. Unpublished manuscript, Science Museum of Minnesota, St. Paul.

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EXECUTIVE SUMMARY

This study was a longitudinal summative evaluation of repeat visitors' experiences in four *Math Moves!* exhibitions that were developed as part of a large collaborative exhibition development project called *Math Core for Museums*, and mounted at four museums around the country: Museum of Science (Boston); Museum of Life & Science (Durham, NC); Explora (Albuquerque); and Science Museum of Minnesota (St. Paul).

A core set of nine exhibit units served as the foundation for each of the four exhibitions, which each institution installed in whatever configuration best served their facility. Many of the museums added additional exhibit units to the exhibitions so they varied greatly in size, layout, and feel. All exhibits were designed to engage visitors in concepts related to ratio and proportion, with an emphasis on kinesthetic understandings and embodied cognition.

The summative evaluation purposively selected four family groups at each institution and collected naturalistic data as the 16 groups engaged with the exhibits from 4-6 times over a two year period. Most initial and final study visits were video recorded with a researcher on site, and all study visits were followed up by an audio recorded face-to-face or telephone depth interview. The primary findings are briefly summarized below.

- Many respondents to the *Math Moves!* exhibitions engaged in a variety of math activities including doubling, size comparison, balancing, matching, x/y axis, and more. But many of them did not think of what they were doing as math.
- The most visited exhibit units were *Rainbow*, *Blocks*, and *Spirograph*. *Clicker*, *Shadow Table*, *Balance*, and *Chairs*, were the next most popular, and *Big Shadows*, *Shapes from Circles*, *Triangle*, and *Comment Board* were the least visited.
- Many respondents tried to use the exhibit challenges presented on the exhibit labels with varying success, and some respondents came up with their own challenges. When they came up with their own challenges, many of the engagements had little to do with ratio and proportion per se, although there was a fair amount of practice with spatial reasoning and other math-related concepts.
- There was a range of group interactions at the exhibits including some adults taking on a traditional teacher role, and some groups were engaged in more playful side-by-side co-learning. When the researchers weren't on site, many adults tended to step back and have the children explore the exhibits on their own.
- A few respondents related things in the exhibits to things they did in school. And some respondents made a relationship between the exhibit and their everyday lives.
- A major focus of the evaluation was if and how respondents increased their fluency with ratio and proportion over repeat visits. Five fluency knowledge hierarchies emerged from the data: fluency with (a) engaging with and using the exhibits; (b) using basic math skills; (c) qualitative and kinesthetic understandings of ratio and proportion; (d) understanding quantitative relationships; and (e) using numbers and other quantitative tools at the exhibits.

- The data indicated that over the two year period, most respondents increased their fluency with engaging with the exhibits, and with their qualitative and kinesthetic understandings of ratio and proportion. Most respondents' fluency with basic math skills also increased, but this was likely primarily due to math skills learned in school. Some respondents increased their fluency with quantitative relationships and using numbers in the exhibits, but to a lesser degree than their qualitative/kinesthetic understandings.
- Two additional hierarchies emerged from the data, hierarchies about (f) the main point of the exhibit, and (g) appreciation for math, ratio, and proportion as part of everyday life. These hierarchies indicated that engagements with the exhibits did not seem to influence most respondents' feelings towards or attitudes about math. As is characteristic of "delayed learning," some influences of the exhibits may not become evident for many years.
- Contributions to increased fluency included (a) the design of increasingly sophisticated challenges, (b) the ability of the exhibit challenges to help visitors discover qualitative and quantitative relationships, and (c) facilitation by adult caregivers.
- Two Visitor Vignettes are presented, each showcasing a different family group. The first is a group that worked collaboratively and intensely during each study visit, paying particular attention to the math in the exhibits. They were able to significantly increase their quantitative fluency with many of the exhibits. The second group also was very diligent in their use of the exhibits but their engagements were primarily qualitative/kinesthetic. Neither the adult nor the child had a strong background in math, and sometimes this got in the way of them increasing their fluency with ratio and proportion, in spite of the diligent efforts of the adult to manage her child's experience.
- Difficulties for respondents included (a) some exhibits that suffered repeated mechanical and maintenance issues, (b) a few design flaws, and (c) some examples of visitors getting side tracked and confused.
- A final section of the report covers respondents' experiences in participating in this project. Visitor respondents appeared satisfied with their experiences although said their participation took more time and effort than they anticipated. Staff respondents gave various examples of ways that participating in the project contributed to their professional development.
- Final lessons learned included the following: (a) the exhibitions appeared to engage respondents in many different mathematical ways including both qualitative/kinesthetic engagements as well as quantitative engagements, including their fluency in a variety of ways; (b) most respondents appeared to increase the sophistication of their engagements with the exhibits over time; (c) staff participants gained important insights about how visitors learn and use math in informal science environments; (d) some staff participants gained important insights about designing for repeat visitors; and (e) adult caregivers played a critical role in helping younger visitors evolve their mathematical fluency.

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1. INTRODUCTION

Museums around the country are increasingly interested in mounting exhibitions about math and math-related concepts. As informal science education has become a mainstay of many communities across the nation and around the world, informal math education is now beginning to gain a foothold as well. One recent venture into the fray is the NSF-funded *Math Core for Museums* (MCM), a collaborative exhibition development and research project among four museums, a university, and a math/science education and research facility.

With a focus specifically on ratio and proportion, the goal of MCM was to develop, create, and evaluate math exhibits that visitors would be able to interact with, at four different museums over multiple visits and over several years, to evolve their understandings of and appreciation for math by immersing themselves in math concepts. In addition to the development of the four *Math Moves!* exhibitions, a concomitant research study into embodied math was also conducted.

1.1 Description of *Math Core* Project

The Project

The *Math Core for Museums* undertaking was a large multi-institution collaborative project to develop a suite of prototype museum exhibits, each exhibit dealing with some aspect of ratio and proportion, and each one incorporating some type of embodied, or kinesthetic, math experience. Using mutually agreed-upon criteria, the project team selected a core set of nine units from the suite of 18 prototype exhibits that were developed. Each of the four collaborating museums received a set of exhibits to serve as the basis for a permanent exhibition at their institution called *Math Moves!*. Each institution configured the exhibits to best fit their museum, and most museums chose to supplement them with additional math exhibits. This is described in more detail below.

The Players

Six collaborating institutions comprised the *Math Core* project: four science museums (Science Museum of Minnesota in St. Paul; Museum of Science in Boston; Explora in Albuquerque; and Museum of Life and Science in Durham, NC); a university: San Diego State University; and an education/research center: TERC, located in Cambridge, MA.

Many individuals participated in the *Math Core* project including museum professionals, administrators, exhibit developers and designers, consultants, educators, researchers, university faculty, graduate students, evaluators, and informal educators. There were a total of 18 core team members, and an additional 12 advisors from many walks of life. Two additional individuals participated in the project as a professional development opportunity. (Appendix A – List of *Math Core* Personnel).

The Process

The basic process for the MCM project consisted of the following components:

In-House Exhibit Development and Prototyping

Each museum developed 4-5 prototype exhibits using their own in-house exhibit development process. A total of 18 prototype exhibits were developed by the four museums, with special attention paid to six pre-determined *desired characteristics* and six *math-related concepts* (Table 1.1), referred to by some team members as "the design principles."

Character
• open-ended, enduring, unlimited, unexpected • evokes and supports math-related conversations • transactive archaeology and physical evidence of prior use • invites and supports parental engagement • accessible & multi-sensory, for everyone • facilitates kinesthetic learning

Math Focus
• decimals • fractions • percentages • ratios • proportions • similarity

Table 1.1: *Character* and *math focus* were important design considerations during exhibit development.

As described in the proposal, an important goal of the project was to develop exhibits that had the potential to engage repeat visitors in ways that would help them evolve their understandings of ratio and proportion as they interacted with the exhibits over multiple visits. As part of the development process, each exhibit team at each museum did its own internal prototyping. Some museums used a prototyping template to guide the process. In addition, an extensive formative evaluation was conducted on selected exhibit prototypes by evaluation staff at the Science Museum of Minnesota (for more details, see section on The Evaluation below).

Teleconferences

Group phone teleconferences were held regularly with the core team members from the four museums, the researchers, and the evaluators to discuss various housekeeping issues, stay in touch with progress and issues at each of the museums, build consensus, brainstorm solutions, and so forth.

Large Group Team Meetings

In addition to the frequently held teleconferences described above, five *large group* meetings were held around the country at the four museums (Table 1.2). These meetings were attended by team members from the collaborating museums, external advisors, researchers, evaluators, educators, and so forth. Rough prototypes of exhibits that were being developed were brought by the museums to many of these meetings. The prototypes were tried out by the team members and visitors; feedback was compiled and later incorporated into design revisions. Additional discussions and presentations about research, evaluation, and related ideas were also part of these meetings.

date	*host organization*	*city*
November 2009	ASTC Conference	Fort Worth, TX
January 2010	Explora	Albuquerque, NM
September 2010	Museum of Life & Science	Durham, NC
March 2011	Science Museum of Minnesota	St. Paul, MN
February 2013	Museum of Science	Boston, MA

Table 1.2: An important part of the exhibit development process, was a series of collaborative large-group team meetings that were held around the country.

Exhibit Selection

After all museums had developed and refined their working prototype exhibits, a series of discussions were held among the team members to select a total of nine units to comprise the core set that each of the four museums would get. This selection was guided by a set of mutually agreed upon criteria (Appendix B – Exhibit Selection Criteria). After the core set of nine units[1] was agreed upon, some museums elected to receive additional exhibits from among the many prototypes, and/or develop/incorporate their own exhibits to supplement the core set.

Of the final set of nine units, seven were common to all four sets; one exhibit was at only three museums; and one was at only two museums. See Appendix C – List of All the *Math Moves!* Exhibits for a complete list of all the units and where they were installed.

Exhibition Installations

Each museum mounted its core set of exhibit units—as well as whatever additional ones they selected—configuring them into an exhibition called *Math Moves!*. Although each museum had at least seven units that were identical, each institution arranged the exhibits to best fit their institution; consequently, each of the four *Math Moves!* exhibitions was different. (See the *The Four Math Moves! Exhibitions* section below for a brief description of each of the four installations.)

[1] Throughout the project, the nine core units were usually referred to as "eight exhibits and a feedback station." This makes sense because the exhibits and feedback station were qualitatively different (see the section *Nine Core Exhibits* for a more detailed description of each unit). Because visitors referred to *all* the units as exhibits, in this report we will do likewise, using the terms *exhibits* and *units* interchangeably.

The Nine Core Exhibits

As noted above, after the first few years of developing prototype exhibit units, nine exhibits were selected by the project partners to form a core set that served as the foundation for each of the four *Math Moves!* exhibitions.

Each of the exhibits (with the exception of the *Feedback Station*) included similar components: (a) a large title panel above the exhibit, followed by a brief one-sentence description of the exhibit; (b) some type of primary kinesthetic or manipulable opportunity; (c) a label that included instructions and "challenges" suggesting things visitors might want to try; and (d) photo montages showing everyday life examples of the mathematical concepts.

All text was in English and in Spanish. Some of the exhibits also had accompanying measuring devices such as rulers or measuring tape, and some had an accompanying bi-lingual audio description—designed to be appropriate for visually impaired visitors—that was accessible via a telephone handset.

An important thing to keep in mind is that the *Math Moves!* exhibits were not designed to formally teach the math concepts of ratio and proportion. Rather they presented challenges that—when accepted by visitors—engaged them in applications of ratio, proportion, and related math concepts. In other words, the exhibits were designed to help visitors learn math skills through *using* math skills.

One of the strategies used to help visitors use math skills, was in the naming of the exhibits. As is supported in the research literature, in this project exhibit titles were thoughtfully and deliberately designed to highlight visitors' action/engagement with math concepts (Perry, 2012). For example, an exhibit using blocks to build structures that were double the size of other structures was titled *Scaling Shapes*. Another exhibit where visitors listened to and graphed various sounds was titled *Sensing Ratios*. Not surprisingly, visitors also came up with their own names for the exhibits. This will be discussed in more detail under *Findings & Discussion: The Role of Exhibit Titles*.

Following is a brief introduction to each of the nine core exhibits.[2] Each unit is identified by its official name, followed in parentheses by the name that visitors called it. For a complete listing of all exhibits that were part of this evaluation study, see Appendix C – List of All the *Math Moves!* Exhibits.

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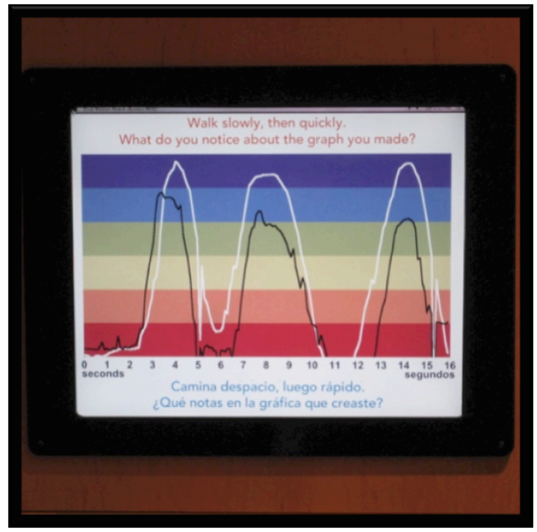
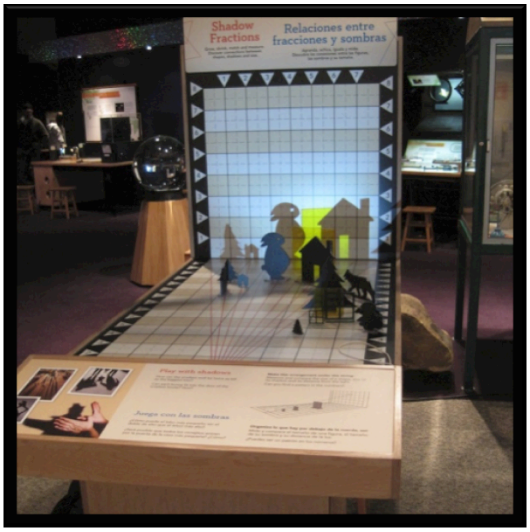
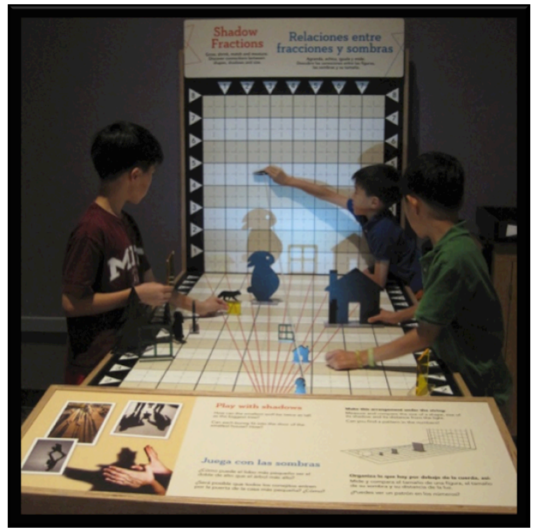
[2] All exhibit descriptions are adapted from the *Math Moves!* website: http://www.mathmoves.org



1. Partner Motion (Rainbow)
This exhibit provides visitors with an opportunity to see how their rate of motion affects a graph on a screen. Walking back and forth, slowly and quickly, visitor pairs create graphs of their motions. The graphs display their movement over time, giving them direct proportional slopes, another way to think about and kinesthetically feel how their rates compare.

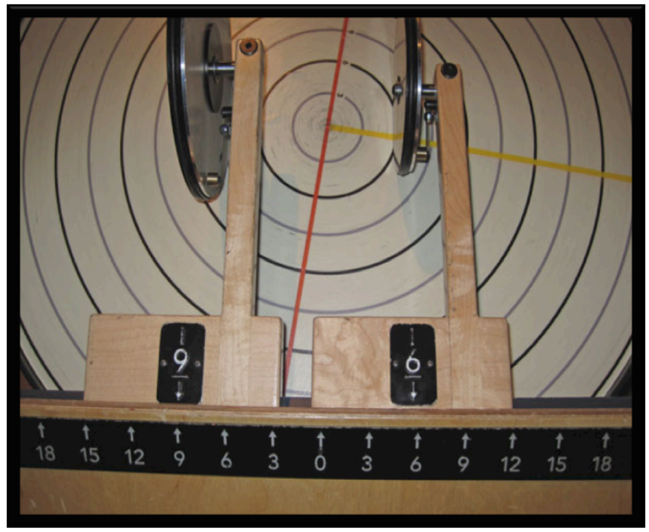
2. Sensing Ratios (Sliders)
This exhibit is designed to explore ratios by hearing tones or sounds, as well as seeing visual representations of them. Visitors move large knobs—at the left and right sides of the exhibit—that control tone generators. The positions of the knobs are graphed on the screen, the left knob on the y-axis and the right knob on the x-axis. A phone at the left supplies an audio description.



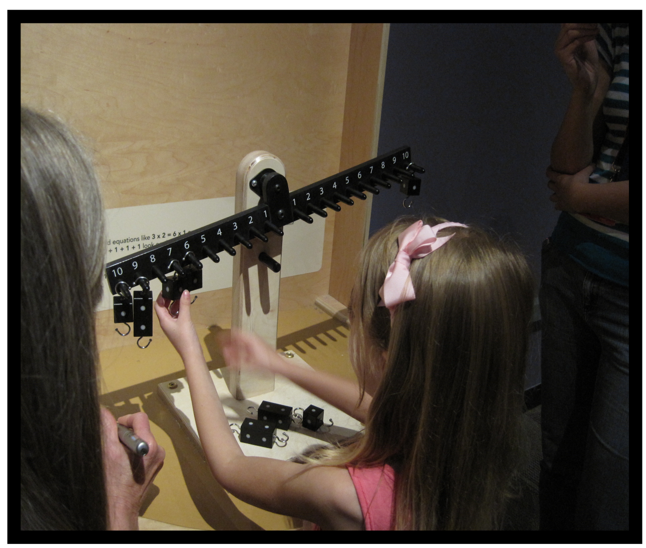


3. Shadow Fractions (Shadow Table)
This exhibit invites visitors to experiment with the placement of scaled objects to make shadow stories or scenes. A bright-white light casts shadows of the objects on a grid. Grid lines are numbered at the sides and along the top. There are three sizes of each object; all are proportional heights: 2” high, 4” high, and 8” high.

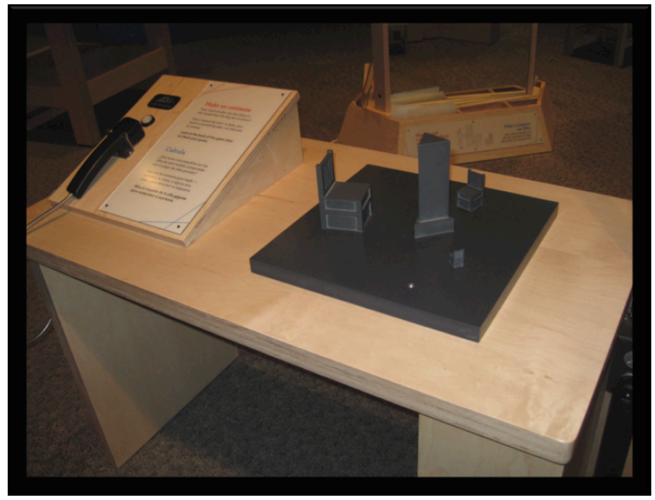
4. Scaling Shapes (Blocks)
This exhibit challenges visitors to enlarge simple objects by doubling all three dimensions. By introducing concepts of volume and three-dimensionality, this exhibit is designed to extend the study of scaling, similarity, and proportionality.



5. Comparing Frequencies (Clicker)
This exhibit creates rhythmic percussive sounds using one or more small wheels driven by a large wheel. Visitors place small wheels against the larger wheel. Each of the small wheels has a clicker that sounds each time the small disk turns once around. The frequency of the clicks depends on the diameter of the small wheel and on where the wheel contacts the large turning wheel. Visitors experiment with several wheels to both see and hear the rhythm of proportions, and the frequency of clicking.



6. Balance and Imbalance (Balance)
This exhibit challenges visitors to hang weights on three types of balances to discover proportions of distance and weights: (a) traditional math education balance, (b) a ratio balance with a circular scale, and (c) a multi-armed balance with a high degree of complexity and open-endedness. This exhibit is designed to help visitors get an intuitive feel of ratios and proportions.



7. Comparing Forms (Chairs)
This exhibit has visitors sit in three chairs that are identical in every aspect except proportional scale. One chair is full scale, one is half scale, and the other is double scale. The exhibit is designed to have visitors use their bodies and other measuring tools to investigate how the chairs differ in size, and to tangibly experience scale, similarity, and proportional relationships between different-sized objects.

8. Drawing with Gears (Spirograph)
This exhibit is a mechanical drawing table where visitors are able to draw harmonic patterns using proportional wheels. The device used several interchangeable gears at a time, one or more of which may be selected by the visitor, similar to the popular Spirograph child's toy. Visitors can produce complex, circular drawings on paper that they can take home or post on the unit.

9. Feedback Station (Comment Board)

This unit has blank cards for visitors to write or draw their concept of double in response to the question: "How did you show double?".

The Four *Math Moves!* Exhibitions

As noted above, each of the four *Math Moves!* exhibitions was unique both in the specific exhibit units included, as well as in the configuration of the exhibits. Following is a brief introduction to each of the four exhibitions.

Explora




Installations at Explora usually include small cubby-like enclosures where visitors can engage intimately with the exhibits. The *Math Moves!* exhibition was installed using this same principle. Many additional similarly themed (but non-*Math Moves!* exhibits) were scattered around the edges of the exhibition. The exhibition was located on the first floor of the museum, away from the front door.

Museum of Life & Science

Located on the first floor of the museum, the *Math Moves!* exhibition at the Museum of Life and Science was installed in a large open space that visitors passed on their way to other exhibitions. A doorway off to the side led to a popular inventing space called *Contraptions*. After most of the data was collected for this study, a few additional *Math Moves!* exhibits were installed in an area on the second floor.

Museum of Science

Located on the first floor of the museum, the *Math Moves!* exhibition was installed in a large deep alcove. At the far back of the exhibition there was a popular virtual fish tank exhibit (unrelated to *Math Moves!*), and along the left wall of the exhibit hall was a portal into a dinosaur exhibition.

Science Museum of Minnesota

Visitors enter the museum on the ground floor (Level Five) and then descend stairs to reach many of the exhibitions. *Math Moves!* was located on Level Three and was easily visible to visitors as they descended the stairs, or exited the elevators. The exhibition was an open floor plan, located against a corner and open on two sides. Adjacent exhibitions included the museum's *Experiment Gallery*.

Supplemental Materials

In addition to the exhibition, a *Math Moves!* website was developed.[3] The website included three main sections: Activities, Exhibits, and Research. Available on the website was also a downloadable, brightly colored, 29-page educator guide with descriptions of the exhibits, definitions of key terms and math concepts, suggested pre- and post-visit activities, descriptions of Common Core *Math Moves!* concepts, and a list of additional resources. Each of the four

[3] http://www.mathmoves.org

hosting museums also included information about the exhibition on their websites, with a link to the *Math Moves!* website.

The Research

An important part of the MCM project, was a concomitant research study conducted by two team members—Dr. Ricardo Nemirovsky and Dr. Molly Kelton—at the Center for Research in Mathematics and Science Education at San Diego State University. This research study was an examination into how people learn math, with a focus on *embodied math* and built on a solid foundation of work that the Center has been doing for many years. The following abstract briefly describes the focus of their study.

> Research in experimental and developmental psychology, cognitive science, and neuroscience suggests that tool fluency depends on the merging of perceptual and motor aspects of its use, an achievement we call perceptuomotor integration. We investigate the development of perceptuomotor integration and its role in mathematical thinking and learning. Just as expertise in playing a piano relies on the interanimation of finger movements and perceived sounds, we argue that mathematical expertise involves the systematic interpenetration of perceptual and motor aspects of playing mathematical instruments. Through 2 microethnographic case studies of visitors who engaged with an interactive mathematics exhibit in a science museum, we explore the real-time emergence of perceptuomotor integration and the ways in which it supports mathematical imagination. (Nemirovsky, Kelton, & Rhodehamel, 2013, p. 372)

Both Nemirovsky and Kelton were active participants throughout the MCM project, engaging team members in numerous discussions of and opportunities for engagement with embodied math experiences. Their work influenced much of the design and development of the *Math Moves!* exhibit units. For example, many of the exhibit developers worked on incorporating kinesthetic opportunities for visitors in such a way as to achieve embodied math outcomes. Their presentations and discussions also helped change how team members thought about how visitors can do math in museum settings in ways that will be discussed in the Findings section, below. A number of papers related to their work are included in the reference list of this report (Hall & Nemirovsky, 2012; Nemirovsky, Kelton, & Rhodehamel, 2012; Nemirovsky & Ferrara, 2009; Nemirovsky, Kelton, & Rhodehamel, 2013).

The Evaluation

An important aspect of the *Math Core* project was the incorporation of evaluation throughout the project. Evaluation efforts included (a) repeated refinement of the exhibit prototypes, conducted primarily in-house by each museum using their own internal processes with additional vetting and gathering of feedback at the regular teleconferences and at each of the large group team meetings, (b) formative evaluation conducted by Dr. Steven Guberman, a Science Museum of Minnesota internal evaluator, on the 18 prototype exhibits (Guberman, Bernstein, King, Onkka, Ostgaard, & Van Cleave, 2011); (c) a critical review of the prototype exhibits to determine how well they carried out universal design goals (Lindgren-Streicher, 2011); and (d) a final summative evaluation of repeat visitor experiences at the four *Math Moves!* exhibitions, conducted by external evaluators Selinda Research Associates.

The primary focus of this final summative evaluation was a longitudinal examination of sixteen recruited families' experiences (four at each of the four museums) as they engaged with the exhibits 4-6 times each from May 2012 – November 2013. The remainder of this report focuses exclusively on this final summative evaluation.

1.2 Tips for Reading the Report

Definitions
In this report we use the terms *exhibit* and *exhibit unit* interchangeably to refer to individual, stand-alone and self-contained modules. The term *exhibition* is used to refer to the specific configuration and set of exhibits that comprised *Math Moves!* at each museum. *Math* in this report is defined very broadly, in keeping with the project team's definition that included kinesthetic and qualitative understandings. (Most visitors on the other hand tended to think of math in more traditional ways, including numbers and operations like addition and division.)

Inclusive Language
Selinda Research Associates is committed to the use of inclusive and non-sexist language. At times in this report this may include the use of *they* and *their* as third person singular. This has a long tradition (Miller & Swift, 1980) and is becoming increasingly accepted in the museum field.

Strength of Number
As described under the Methodology section below, this evaluation study was grounded in naturalistic inquiry. We have found that the inclusion of statistics in a naturalistic report can create a false impression of specificity or strength of finding. Instead, in accordance with standards for naturalistic inquiry, we use the adjectives *most*, *many*, *some*, *few*, and *none* to describe tendencies.

> In reporting aggregate information such as interview data, we made the decision not to tabulate percentages of different kinds of responses offered by visitors. It is our view that once numerical ratings are assigned to judgmental data, there is a great tendency on the part of the reader to engage in hyperbole and misinterpretation. Rather, we have intentionally used adjectives such as "most," "many," "some," "few," or "none" to help portray aggregate tendencies. If we simply say visitors, then it implies that almost everyone was in agreement (Wolf & Tymitz, 1981).

Reporting the Findings
In many evaluation reports, findings are reported by data type. In other words, interview findings are presented separately from observation data. In naturalistic evaluation, the aggregate of what was found is primary, regardless of the source. In this study topics and issues emerged from the data and are reported as such. All direct quotes from respondents are followed by a brief data

code indicating its source, e.g. (SC4) or (D6-4). All codes that begin with *SC* refer to staff interviews; *A*, *B*, *C*, and *D* refer to recruited respondents.

Names of Exhibits

Throughout the project as the exhibits evolved over time, they were referred to by a variety of different working titles. As noted above, the final selection of exhibit titles by the project team was a deliberate attempt to focus visitor attention on the mathematical operation. But visitors often came up with their own names for the exhibits. Because this report is about articulating visitors' experiences in *Math Moves!*, in this report all exhibits will be referred to by the names that respondents tended to use when they talked about them. This is not meant to denigrate the deliberate and thoughtful work of the exhibit developers but rather to ensure the report accurately reflects the visitor experience. Most of the exhibits discussed in this report are from the core set of nine described above, but many others will also be mentioned. For a complete list of all exhibits, their official names, and visitors' nicknames, we suggest the reader bookmark and frequently refer to Appendix C – List of All the *Math Moves!* Exhibits.

2. OVERVIEW OF THE SUMMATIVE EVALUATION STUDY

The summative evaluation began with extensive collaborative development of a detailed evaluation plan. This evaluation plan document is included in Appendix D – Evaluation Plan. Throughout the project, it served as the blueprint for the evaluation study, and was repeatedly referred to. This section of the summative evaluation report presents a brief overview of the study; for additional detail, the reader is referred to Appendix D – Evaluation Plan. In a few instances, slight deviations from the evaluation plan were necessary. These are described below. The reader should note that whenever there is a discrepancy between what is written in the body of the report and the evaluation plan, the body of the report is the most up-to-date and accurate account. During the course of the project, three sub-groups played critical roles in various aspects of the evaluation: the *Math Core Project Team* (MPT), the *Core Evaluation Team* (CET), and the *Museum Rep Team* (MRT). Each of these teams is described in detail in Appendix D – Evaluation Plan.

The Research Questions

The primary research question for this evaluation study was:

> In what ways and to what extent do the four *Math Moves!* exhibitions contribute to the development of repeat visitors' understanding of, appreciation for, and fluency[4] with ratio and proportion (and related math concepts) as they recurrently engage with the exhibitions over two years?

A secondary question related to the contribution of MCM to professional development:

> In what ways and to what extent did participating in this project contribute to the evolution of museum professionals' thinking about (a) the development of math-related STEM visitor experiences; (b) the development of ISE opportunities for repeat visitors; (c) their own understandings of and relationships with concepts related to math, ratio, and proportion; and (d) the power and efficacy of collaborative projects.

In order to answer these research questions, a detailed Topical Framework was collaboratively developed. This framework outlined many issues to explore, and is included in Appendix E – Topical Framework.

Methodology

In research reports, methods and methodology are often confused. In this report, we adhere to Harding's distinction as described in the following quote:

[4] Note that *fluency* in this case does not refer to complete mastery, but rather a sense of being comfortable and at ease.

> A research method is a technique for (or way of proceeding in) gathering evidence.…A methodology is a theory and analysis of how research does or should proceed." (Harding, 1987, p. 2)

In this study, a naturalistic *methodology* was used. Naturalistic inquiry is a rigorous and systematic approach for collecting and analyzing data in real-life settings. The goal of naturalistic methodology is to provide a holistic understanding of participants' experiences from a variety of perspectives and using a variety of *methods* (Lincoln & Guba, 1985). In this case it included collecting both qualitative and quantitative data from a variety of sources and triangulating the data to develop a thorough understanding of the various participants' experiences. One of the strengths of naturalistic evaluation is that unanticipated findings often emerge from the data, often in participants' own words. This type of inquiry allowed the researchers in this study to follow up on threads and themes that characterized how respondents thought about their experiences.

Naturalistic inquiry is guided by a different set of criteria than experimental or positivistic research. In judging the quality of a particular naturalistic study, constructs such as credibility, transferability, dependability, and confirmability take the place of more familiar constructs such as reliability, validity, and generalizability (Allen, Gutwill, Perry, Garibay, Ellenbogen, Heimlich, *et al*., 2007). Every attempt was made to ensure this study adhered to the highest standards for naturalistic evaluation.

It should be noted however, that, while naturalistic inquiry usually focuses on naturally-occurring real-life settings, in this study, the recruited respondents—the primary source of the data—were highly motivated in ways that more naturally-occurring museum visitors would not be. In all other aspects however, the study was grounded in the underlying tenets of naturalistic inquiry including (a) axioms about the nature of reality, the interaction between the knower and the known, the possibility of causal linkage, the role of values in inquiry, and the possibility of generalization (Rau, 1990, p. 39-54), and (b) established standards for conducting naturalistic inquiry, including credibility, transferability, dependability, and confirmability (Allen, et al., 2007, p. 236-239)].

Overview of Study Design & Methods

The summative evaluation study was conducted in five broad (and often overlapping) phases as outlined below in Table 2.1. Four family groups – i.e. an adult and a focus child – were recruited at each of the four museums to participate in the study. Two Selinda researchers were each assigned to the eight family groups at two museums for the duration of the study. In addition to their assigned Selinda researcher, each group also had a local museum contact.

Each family group made a total of 4-6 study visits over an 18-month period of time. During the study visits, the groups used the *Math Moves!* exhibitions however they wished, and were then interviewed by a trained researcher.

During the *initial* and *final* study visits, the Selinda researcher was on site and observed the family group's use of the exhibition. These initial and final study visits were audio- and video-recorded. Immediately after the respondents finished visiting the exhibition, the researcher conducted a post-visit depth interview with the family. These post-visit interviews were also audio- and video-taped. At one museum (Explora) the final study visits were not able to be done in conjunction with an on-site visit by the researcher so these visits were videotaped by the local museum representative and immediately followed up by a phone interview with the researcher.

During the *interim* study visits, each family group used the exhibition by themselves. They then arranged to have a telephone or Skype post-visit interview with their assigned researcher within a week or two of their visit. All interim post-visit interviews were audio-recorded.

In addition to observing and interviewing the recruited family group respondents during their site visits, the assigned researchers also conducted a few unobtrusive observations of purposively selected regular museum visitors using the exhibitions. And one of the Selinda researchers also conducted nine depth interviews with purposively selected *Math Core* project staff.

phase	*approximate begin*	*approximate end*
Immersion in the *Math Core* project and culture	October 2009	July 2014
Collaborative planning for the evaluation	May 2011	May 2012
Data collection		
casual respondents	March 2012	November 2013
recruited respondents	May 2012	November 2013
staff respondents	August 2013	January 2014
review of documents	March 2012	January 2014
Data analysis	May 2012	August 2014
Report writing	March 2014	March 2016

Table 2.1: The summative evaluation took place in five broad overlapping phases.

Data collection consisted primarily of depth interviews, both unobtrusive and participant observations, and document review. An important part of data collection was that after each interview, observation, or review of a document, the researcher immediately after (or as soon as possible) prepared a detailed written debrief fleshing out their notes, summarizing the important findings, reflecting on the results, and preparing for the next interview, observation, or document. These written debriefs were shared among the three researchers and the findings were discussed during periodic group debrief sessions throughout the project. The methods and the overall study design are described in more detail in Appendix D – Evaluation Plan.

The Respondents

As described above, this study included three types of respondents: (a) recruited family group respondents (RRs), (b) casual visitor respondents (CRs), and (c) museum professional respondents (MPRs). Each of these is described in detail below.

Recruited Respondents
The primary respondents for this study were 16 purposively selected recruited respondent groups, four at each museum. Each group was comprised of a *focus child* and an *accompanying adult*, although additional family members and/or friends often accompanied the core group during the study visits. Two of the 16 groups had two focus children (twins), for a total of 18 focus children in the study. In a few instances, due to logistical issues an alternative adult accompanied the focus child. See Appendix F – Description of Recruited Respondents for a brief overview of each RR group.

All RRs were recruited and purposively selected by the museums to include a wide range of diverse families including age of focus child, family configuration, languages spoken at home, experience and fluency with math, attitudes toward math, etc. The children's ages ranged from 5 through 11 years old at the beginning of the study, and they were in kindergarten through fifth grade. There were a total of 10 girls and 8 boys. (See Tables 2.2 and 2.3.)

age at 1st visit:	*5yo*	*6yo*	*7yo*	*8yo*	*9yo*	*10yo*	*11yo*
# of RR girls	2	1	4	0	2	0	1
# of RR boys	0	1	0	2	1	2	2
Total (n=18):	*2*	*2*	*4*	*2*	*3*	*2*	*3*

Table 2.2: The RR focus children spanned a range of ages.

grade at 1st visit:	*K*	*1*	*2*	*3*	*4*	*5*
# of RR girls	3	2	2	0	2	1
# of RR boys	1	0	1	1	1	4
Total (n=18):	*4*	*2*	*3*	*1*	*3*	*5*

Table 2.3: The RR focus children spanned a range of grades.

Children	*high*	*medium*	*low*
math knowledge & experience	7	7	4
attitudes toward math	8	9	1
Total (n=18):	*15*	*16*	*5*

Adults	*high*	*medium*	*low*
math knowledge & experience	4	5	7
attitudes toward math	6	4	6
Total (n=16):	*10*	*9*	*13*

Table 2.4: RRs included a wide range of self-reported experiences with and attitudes towards math. Note that *knowledge and experience* is in relationship to age. One child was described as medium-high in both knowledge & experience, and in attitude. This was counted as medium in the table.

Recruited respondents for this study (both adults and children) also included a wide range of knowledge and experience with math, as well as a range of attitudes towards math. During the initial recruitment/selection interviews with adults, candidates were asked to describe their background knowledge, experience with, and attitudes towards math, and do the same for the focus child. Table 2.4 provides a quick snapshot of how respondents self-reported.

Recruitment and Selection

Each museum representative did the initial recruitment and selection of possible family group respondents for their museum. The recruitment process is described in detail in Appendix D – Evaluation Plan, Appendix G – Host Venue Instructions, Appendix H – Sample Recruitment Flyer, and Appendix I – Recruitment Information Form.

After interviewing many potential respondents, the museum representative submitted a short list of 5-11 names for consideration to the CET (Core Evaluation Team). A total of 32 names were submitted from the four museums. Using purposive sampling technique,[5] the CET reviewed the completed Information Forms, discussed each candidate, and finally selected four RRs from each institution. The museum reps then contacted the selected candidates to determine if they were still interested and to make sure they understood the commitment required. A few declined to participate, so the next group on the list from that institution was invited. All final recruited respondents completed a Consent & Agreement Form (see Appendix J – Sample Consent & Agreement Form). This form was identical across institutions, except for the letterhead, name and contact information of the museum rep, and the description of the tokens of appreciation, which varied by institution. (For a complete description of tokens of appreciation offered by each museum, see Appendix K – Tokens of Appreciation by Institution.)

The recruitment and selection protocols, and consent and agreement forms were submitted to Museum of Science's IRB and approved as Protocol 2012-02. Periodic renewals were obtained as necessary.

Casual Respondents

In addition to the RRs (Recruited Respondents) described above, it was important to observe and interview some visitors who were not recruited ahead of time, but who had chosen to visit the exhibitions on their own, i.e. Casual Respondents (CRs). While CRs were not a primary source of data, Selinda researchers spent an additional 15 hours in the exhibitions (beyond the time they spent with RRs) observing and interviewing approximately 200 individuals in 90 visitor groups.[6]

During these observations and interviews, special attention was paid to four different types of engagements: physical, social, emotional, and intellectual (Perry, 2012). Casual respondent data

[5] See Appendix D – Evaluation Plan for a brief description of purposive sampling.

[6] These figures represent a conservative estimate of the actual number of CR hours, individuals, and groups. While most CR observations and interviews were fully documented, sometimes the exhibitions were too crowded and/or researchers' time was spread too thin to fully account for all observations and interviews. In these cases, estimates were made.

was collected whenever the Selinda researchers were on site, primarily during the initial and final RR study visits, but also to a limited extent during the large group team meetings mentioned above.

Staff Respondents
Out of the approximately 30 staff participants in the MCM project (18 core team members and 12 advisors), nine were purposively selected to include a wide range of perspectives. Each of these individuals was invited to participate in a depth telephone interview conducted by an SRA researcher. A total of nine depth interviews were completed with the MPRs (Museum Professional Respondents), with interviews ranging from 35 minutes to 66 minutes. After each depth interview, a detailed debrief was written, for a total of approximately 30-35 contact hours. In addition to the depth interviews, the lead Selinda researcher served as a participant observer during many of *the Math Core* staff teleconferences, as well as most of the large group team meetings.

The Researchers
Three Selinda researchers participated in this study: Dr. Deborah Perry (Director of Selinda Research Associates) was the lead researcher. She oversaw and managed all aspects of the evaluation study, and participated in many site visits at each of the four museums including nine of the 16 initial RR study visits.

Dr. Eric Gyllenhaal (Senior Research Associate at Selinda Research Associates) was the lead researcher at Explora in Albuquerque (EXP) and Science Museum of Minnesota in St. Paul (SMM). Diane White (now Zinni) (Research Associate at Selinda Research Associates) was the primary researcher at Museum of Science in Boston (MoS), and Museum of Life & Science in Durham, NC (MLS). Each museum had a Museum Representative, a staff member who served as the primary contact person for the RRs, and liaison with the researcher.

The Study Visits
Recruited respondents were initially responsible for visiting the *Math Moves!* exhibition a total of at least six times over the course of the study. All initial and final study visits were scheduled over a two-day period so that their assigned SRA researcher could be on site and observe their engagements first hand. The four interim study visits were to be scheduled at each RRs convenience, although the Museum Rep and researcher were informed prior to the visit so that parking and other arrangements could be made, and the follow-up interview scheduled.

Towards the end of the second year of the study, it was clear that some RRs were finding it more difficult to schedule their visits, and that some of them were losing interest. Because it was also determined that a very large amount of data had already been gathered, the Core Evaluation Team (CET) decided to give RRs who had not yet scheduled their remaining interim visits, the option of doing only two or three interim visits—an overall total of four or five visits instead of the original six. Three of the 16 groups completed a total of four study visits; five completed five study visits; and the remaining eight completed all six. It should be noted that most of the RRs that completed all six visits, had either already completed or scheduled their fifth interim visit by the time the decision was made to make the remaining interim visits optional. One family (RR13) was not able to complete a final study visit as described above, because they moved shortly after their fourth study visit. And one family made a few additional non-study visits to the exhibition. In other words, they visited *Math Moves!* on their own without contacting the museum ahead of time, and without doing a follow up interview with the researcher. (See Table 2.5 below for a summary of all study visits by all RRs).

	May	June	July	August	September	October	November	December	January	February	March	April	May	June	July	August	September	October	November	# of study visits
RR1		x						x				x				x				4
RR2		x					x		x			x					x			5
RR3		x			x			x			x				*	x				5
RR4		x				x		x								x				4
RR5		x				x	x		x					x					x	6
RR6		x				x		x		x				x					x	5
RR7		x				x		x				x							x	5
RR8		x			x	x				x				x					x	6
RR9		x			x			x			x			x				x		6
RR10		x	x					x			x				x			x		6
RR11		x			x			x				x			x			x		6
RR12		x			x		x				x				x			x		6
RR13	x	x		x								x								4
RR14	x	x			x						x			x		x				6
RR15	x				x			x			x					x				5
RR16	x				x			x		x				x		x				6
Total number of study visits by RRs:																				*85*

Table 2.5: Recruited respondents made a total of anywhere from four to six study visits to the *Math Moves!* exhibitions over a period of time that ranged from 12-18 months. One RR family (RR13) did not do a final study visit because they moved away during the study.

Initial Study Visits

For the initial study visits, each of the RR families was scheduled to visit the museum at a pre-determined time over a two-day weekend. At their assigned time, the Museum Rep and the researcher assigned to that museum met the group and introduced them to the study. They were then taken to the *Math Moves!* exhibition and invited to use the exhibits however they wanted, and for as long as they wished. During the time they engaged with the exhibits, the researcher observed them, taking notes about their interactions. In some cases the researcher engaged with them, asking questions about what they were doing, or thinking about. During their time in the exhibition, all RR (Recruited Respondent) interactions with the exhibits were recorded (both audio and video). Some additional photographs were also taken. When the respondents indicated they were finished engaging with the exhibits, they were taken to a separate room where they participated in an interview with the researcher who asked them to elaborate on some of the things they were doing and what they were thinking. A second SRA researcher was also part of many of these initial study visits.

Interim Study Visits Two – Five

All interim study visits by the RRs took place without the researcher on site. What usually happened was that the RR contacted the Museum Rep and informed them that they would be coming to the museum as a study visit on a particular day. The RR group would then visit the museum and spend as much time with the *Math Moves!* exhibits as they wanted. They would not be observed during these interim visits, but some adult RRs took notes of what they and their children did. As soon after the study visit as possible, the RR family group would have a telephone or Skype interview with their assigned researcher. They were also encouraged to send photographs or drawings of their visit if they wished, and many did.

Final Study Visits

At three of the four museums, the final study visit was similar to the initial one, with a researcher on site, the focus child wearing a microphone, and the family group videotaped. The primary difference between the initial and final study visits was in the directions that RRs were given by the researcher. During the initial visits they were encouraged to explore however they wanted, whereas in the final study visits they were invited to give the researcher a tour of the exhibition.

In accordance with standards for conducting naturalistic inquiry, instructions given to different RRs varied depending on the specific family group. An example of the type of instructions given respondents is as follows: "You've been to the math exhibits at least five times so far, but this visit is going to be a little different. Sally, you're going to take me on a tour of some of the exhibits and show me what kinds to things you can do with them. We will also have a video camera follow along, so I can remember what you did, and you'll have a microphone on, so we'll have a recording of what you say. We will try to spend at least a little time at each of the exhibits. We want you to both show me the kinds of things you do at the exhibits, and also talk about what you are doing. I may have a few questions as you go along, and I may remind you to talk about things as you do them. Sally, we'd like you to show us each exhibit first. Then, if mom or dad wants to show or tell us something more about that exhibit, they can talk after. Okay?"

At one of the four museums, logistics prevented the SRA researcher from being on site during the final study visits. At this museum, the researcher worked with museum staff to have each RR's final visit taped by the Museum Rep who both operated the video camera and sometimes asked questions or made suggestions about activities to try. These final study visits were scheduled over a three-week interval at times that were convenient for the RRs, but also coincided with SRA researcher's schedule so that a final Skype video interview (set up by the museum) could take place immediately after the RRs' time with the exhibits.

Limitations

As with any study, this undertaking was limited in a number of ways. This was an extremely ambitious evaluation study, one that generated a huge amount of rich and interesting data. Following 16 family groups over 18 months as they visited four different museums multiple times was a major undertaking, one that was necessarily constrained by limited time and money. As such, its findings must be taken in context.

Most of the data for this study came from respondents who were specifically recruited and compensated for their significant investment of time and energy. These respondents were highly motivated and dedicated to the study; their engagements were not typical of casual visitors. In other words, their experiences paint a picture of possibility rather than generalizability.

All interim study visits by RRs (recruited respondents) were self-report only, with a few accompanying photographs and/or drawings. Due to the resources available, no external triangulation was done. For example, if they said they spent half an hour in the exhibition, and did *Rainbow* and *Sliders*, or that their favorite exhibit was *Chairs*, we took their word for it, and did not make any attempt to externally validate this. Triangulation was done more extensively during the initial and final study visits because the researcher and at least one or two additional museum staff were on hand.

Member-checking is an important aspect of many naturalistic studies. This is when the researcher shares the written debrief with the respondent to make sure the debrief accurately reflects what they observed and heard. Due to resource limitations, member-checking in this study was limited to the researcher periodically reflecting back to the respondent during the interview or observation what they thought they were hearing or seeing and asking for clarification.

Although rarely undertaken in exhibit evaluations, another important aspect of naturalistic inquiry is an external dependability audit. This is when someone external to the study reviews the audit trail of the researchers, including all the activities, field notes, and written debriefs, to assess credibility and transferability. (Allen, Gutwill, Perry, Garibay, Ellenbogen, Heimlich, et al., 2007). In this study a dependability audit was not conducted due to resource limitations.

This study was an effort to tease out what, if any, contributions going to *Math Moves!* four-to-six times over an 18 month period of time made to family groups' understandings of, attitudes towards, and fluency with ratio and proportion. It must be noted that many factors external to the study – including the children's natural maturation, development, and educational experiences – also contributed in significant ways that are beyond the scope of this study.

Finally, unlike many evaluation studies that take place over a relatively short period of time, in this study repeat visitors' experiences were explored for many months. That said, it is still likely that many of the long-term contributions of repeatedly engaging with the *Math Moves!* exhibits will not be fully realized for years, or even decades. In one telling example from a study conducted in the 1990s the respondent described in detail how he didn’t fully understand or appreciate his childhood repeated engagement with one particular exhibit at the Lawrence Hall of Science until many years later when he was taking a college physics course. Those early repeated engagements with the exhibit turned out to have a profound effect on his later understanding of important physics concepts. It may turn out that 18 months is simply too short a period of time to notice many of the important contributions of the *Math Moves!* exhibitions (Perry, 2002).

3. FINDINGS & DISCUSSION

An initial question about visitors' experiences in a museum exhibition is often "Did they enjoy themselves? Did they enjoy interacting with the exhibits?" The data from this study clearly indicated that many visitors to the *Math Moves!* exhibitions enjoyed their time in the galleries and their time engaging with the exhibits. Many of the recruited respondents (RRs) in particular quickly identified units they declared their favorites, and demonstrated this by repeatedly returning to them during each study visit.

The *Math Moves!* exhibits were carefully designed to engage visitors with the math concepts of ratio and proportion in different ways, even when it was not obvious to the visitor that that's what they were doing. Most respondents therefore experienced math at most exhibits simply by engaging with them as they were designed, oftentimes without thinking specifically about math but nonetheless being engaged with math concepts. Visitors racing each other at *Rainbow* noticed that how they moved changed the graph; moving the figures at *Shadow Table* made the shadows get larger or smaller; and placing the *Clicker* wheels at different positions on the spinning disk created faster and slower clicks. By moving one's body and manipulating exhibit objects, visitors embodied many math concepts without even being aware they were doing math, for example, gradually figuring out over time how to move their arms to draw a graph or a "picture" on the screen at *Sliders* or the relationship between a high and a low pitched sound; learning just where to place cubes to balance the beam at *Balance*; or realizing that different sized wheels made different types of drawings at *Spirograph*.

There were some respondents however who tended to use the exhibits in ways that were not particularly focused on math, for example using the figures at *Shadow Table* to create stories or plays that didn't involve the shadows; using *Spirograph* to draw beautiful pictures while paying little attention to the gears; or building (and knocking down) tall towers at *Blocks*. It was not unusual for *Chairs* to be used primarily as a photo opportunity; and a few respondents were observed using the weights at *Balance* to make a necklace or bracelet.

As will be described in more detail below, many of the RRs used the exhibits to evolve their engagements with, understandings of, and fluency with math and math-related concepts over time, at least to some degree. Some respondents were more successful at this than others. Even after numerous visits there were a few RRs who preferred to engage with the exhibits in primarily non-math ways, such as are described above. By their final study visits however, some RRs were engaging with ratio and proportion in varied and sophisticated ways at different exhibits. This was in comparison to casual respondents; only a few CRs engaged with similar sophistication.

The remainder of this chapter presents detailed findings from the study.

3.1 Where's the Math?

One of the challenges in developing exhibitions such as *Math Moves!* – exhibitions that are designed to seamlessly and kinesthetically engage visitors with math – is the question *If visitors don't recognize that what they are doing is math, is it still math?* (Wright & Parkes, 2015).

There was strong evidence that many of the visitors to these exhibitions – while engaging in rich and meaningful math experiences – did not see that what they were doing was about math. The following section explores this issue from two angles: first, what counts as math in general, and then secondly, visitors' perceptions of the math in the exhibitions.

What Counts As Math?

One of the challenges of this project was figuring out what counts as math. While the project team consciously and explicitly had a consensus view of math for *Math Core* (i.e. a broad definition of math experiences of ratio and proportion – a definition that included body movement, and one that did not limit itself to classroom math), many ways of thinking about math emerged throughout the project, from a traditional view of math as derived from what is learned in the classroom, e.g. "widely circulating notions in our culture about what math is…stereotypes of math" (SC4); to a broad view of math as ideas and a way of thinking, e.g. "math is ideas and a way of experiencing the world, and there is a value in that" (SC2); to notions of embodied math, for example this description of the exhibits from the *Math Moves!* educator guide:

> *Math Moves!* is about experiencing ratio and proportion. Allowing students to play with ideas of ratio and proportion in tangible ways provides a physical memory and background as a basis for developing abstract patterns, associations, and concepts. This qualitative sense of ratio and proportion can support quantitative competence when they encounter ratios in classroom work. (Science Museum of Minnesota, n.d.)

The MCM exhibits were deliberately designed to include math by focusing on such concepts and skills as ratio, proportion, fractions, percentages, similarity, measuring, spatial reasoning, and reading/using graphs. But it also was important to the design that the math wasn't too "heavy-handed," i.e. that visitors would be immersed in math, and may even be able to eventually recognize that they were doing math, but not be overwhelmed by the math.

> We wanted to develop people's intuitive, informal notions of ratio….We also hoped to give people a physical memory that involves playing with ratio and proportion so that later when they encounter more formal notions, they could make a connection to this experience. (Wright & Parkes, 2015)

The intention appeared to be that while visitors might begin engaging with the exhibits without paying much attention to the math, they would soon recognize that math was an important focus. As one staff respondent explained:

> We added a lot of elements to these exhibits that made it sort of obvious that there was math going on. There are a lot of graphs. There are a lot of grids. There are a lot of numbers. When you walk through there I think…if you give much thought to it, it's not much of a leap to go: "All this stuff is about math." Because there's a lot of math tools there. There are rulers, there are fractions, there's just a lot of tools that can be used by visitors, or not used by visitors; but they are available, and I think visually they just communicate to visitors that this is about math. (SC2)

One staff respondent talked further about the importance of incorporating a broad definition of math, as described in the following debrief.

> This respondent said they advocated looking at the math in *Math Moves!* broadly, and that they saw that as "one of the huge advantages of these sorts of free-choice learning environments. The potential to engage with a sort of unintended sub-domain, like spatial reasoning, and things related to interpretation of graphs, for example. I think that's one of the huge advantages of these sorts of spaces." They talked about letting visitors' experiences take the lead in telling us what kinds of mathematics they might be engaged in. "It's really not restricted to engagement with ratios and proportions." (SC4)

Defining Ratio and Proportion

In spite of many discussions about what constitutes math in general, the *Math Moves!* exhibitions were focused specifically on experiences that use ratio and proportion. As explained in the educator guide, a **ratio** is "a comparison of two things," and **proportions** are "statements that two ratios are equal. In proportion problems there are two things that both change at the same rate"[7] (Science Museum of Minnesota, n.d., p. 3).

When thinking about ratio, a comparison of two things, the simplest comparisons are done verbally, without numbers—one thing is bigger or longer or heavier than the other. You can then ask if something is *much bigger* or a *little bigger* than another thing. In other words, noticing a difference implies that there is a ratio. These qualitative comparisons can become increasingly sophisticated. For example, you can also notice (without necessarily verbalizing it) that the ratio can be big (corresponding to a big difference) or small (corresponding to a small difference). And then, you can start noticing changing ratios. All of these are done without quantizing, and yet all are still ratio and proportion; all are still mathematical.

At some point, you might also use quantitative tools like counting and measuring. For example, when you measure both things, you can compare the numerical results, and you can answer the *how much* question—how much bigger or longer or heavier?—more precisely.

[7] Note that colloquial uses of the term *proportion* are often different than the mathematic usage. For example, the first definition in a *Webster's Collegiate Dictionary* defines a proportion in a way that sounds similar to a ratio: "the relation of one part to another or to the whole with respect to magnitude, quantity, or degree." The third definition in the dictionary is the same as the mathematical one given in the educator guide.

As pointed out in the educator guide, quantitative ratios can be expressed using words, like "double" or "twice as much," or as numbers, such as fractions, decimals, percents, or with a colon between the two numbers being compared. In *Math Moves!* you can see that one chair is *taller* than the other, and you can use the tape measure to tell *how tall* each chair is. And then you can compare the two measurements and find out that one chair is *double or twice the height* of the other, one seat is 1/4 (or 25% of) the area of the next larger seat, or the volume of the largest chair is 64 times the volume of the smallest chair (because it's 4 times taller, 4 times wider, and 4 times deeper = 4x4x4 = 64).

Visitors Seeing the Math

Even though the exhibits were designed to include various manifestations of math as described above, the data indicated that most visitors did not readily see math in most of the exhibits, or think of what they were doing in the exhibition as math. One adult RR in particular described previous visits to the exhibition—before they became involved in the study—and explained that until the first study visit, they did not realize they were in a math exhibit. The respondent explained that this was not a problem, and was perhaps even a benefit.

Even when they were intellectually aware that the whole exhibition was about math however, the recruited respondents still faced the challenge of finding and recognizing math at the individual exhibits. Most adults said they found math at at least some of the exhibits during their initial study visits, although their children—especially their younger ones—often had trouble thinking about or recognizing that they were using math. One adult explained that, before their third study visit, their child asked specifically where the math was in some of the exhibits. They then spent their third study visit deliberately looking for math in the exhibits where the child had not seen it before. It appeared that they ultimately felt successful at most of them, but it should be noted that this experience was a highly motivated respondent group, because they were in the study. It is unlikely that many visitors visiting on their own would make as concerted an effort.

In addition to the challenge many RRs experienced seeing math in the exhibits, there was the even more difficult challenge of finding ratios and proportion. A few of the respondents figured it out pretty quickly. As one adult RR said during the interview after the initial study visit:

> It's math. It's math. Relationships, ratios, things like that; more concepts of math, like increasing the size, and just ratios, proportions, that kind of thing. [It's also about] geometry, the shapes. (A2)

And a few adult RRs appeared to figure out this focus on ratios after multiple visits (A5), and (like the respondent above) some of these even used the term ratios. Some RRs talked about math at some of the exhibits, and many even did doubling, but most of these respondents did not appear to see the exhibits in terms of ratio and proportion.

One notable exception was one very excited 9 year old girl who, after her fourth study visit, shared her new-found discovery with the researcher.

> I learned something really important today. I learned that the whole thing is about octaves and ratios! Well not octaves, but ratios of different sorts. 'Cause they can be audio ratios like at the…octave thingy where you move them up and down and you can hear the different ratios. And then they can be weight ratios, like where you look at like different weights on each side and they tell you you have two to one, and two to one, and then…at the part where you have the blocks and you try to double them. That's a ratio too, you're making it bigger like, times two. And then also the chairs are ratios to each other because the smallest chair is one-fourth the size of the biggest chair, and the medium chair is in the middle. And also the shadows are ratios too! Because you put them together and…the small ones are in proportion to the bigger ones, and also, guess what? The amazing sound clickers are also ratios!...I was thinking about it, and then all of a sudden I realized…that the numbers were going the same in all the different [exhibits] and then I was like, "Oh my goodness. The whole thing is about ratios!" And then it just sort of boinged—it hit me. (D6-4)

Visitors Using Numbers

As noted previously, the *Math Moves!* exhibits were designed to engage visitors with ratio and proportion, even when they weren't using numbers. On some exhibits in particular, like *Rainbow*, numbers were deliberately made "small and unobtrusive" so that they would not get in the way of the fundamental experience (J. Newlin, personal communication, February 13, 2015).

At other exhibits, like *Balance* and *Chairs*, numbers tended to be used by many respondents when they engaged with the exhibit to do things like measure, count, and multiply. These exhibits were often recognized by respondents as being about math. *Blocks* also seemed math-focused to those RRs who did the doubling exercises, but not so much to those who just built cool structures with the blocks. Some respondents said they recognized the doubling activity as math, but then quickly became bored with that singular approach to the blocks, and so reverted to more imaginative (non-math) play.

In terms of the two graphing exhibits, *Sliders* reminded many RRs (both children and adults) of the X-Y coordinate systems they had done in math class. Most appeared less likely to consider *Rainbow* to be about math, probably because, even when they successfully did the illustrated challenges, many respondents did not focus that much on the numbered (in seconds) horizontal axis, and the vertical axis did not remind them of anything they had done in math class.[8]

Two exhibits—*Spirograph* and *Clicker*—had many numbers on the pieces, but these numbers were rarely used to meet the exhibit challenges. For example, some RRs used the numbers that were printed on the *Spirograph* gears as labels (Figure 3.1), but none recognized them as numbers and ratios of gear teeth. One example of this was when a respondent asked another visitor to please "give me disk 30," referring to the gear that was labeled as having 30 teeth. But

[8] It was clear that visitors using *Rainbow* were experiencing math, and even using math language such as faster, slower, up, down, same as, angle etc. Some respondents specifically related their drawings to both time and distance travelled. However, even though they were engaged with math, most RR did not recognize their experience as math.

this respondent never used the word *teeth*, or talked about the ratio displayed on the gear—in this case 4:1 (C3-1).

During the final study visit, the researcher pointed out the gear ratios printed on each disk to one family, and made some suggestions about how they might use them. The (11 year old) child in the family explained that she had known about the concept of ratios for a while from school, and remembered seeing the numbers (separated by colons) on the wheels at *Spirograph*, but didn't think ratios had any connection with what she was doing at the exhibit (C2-6).

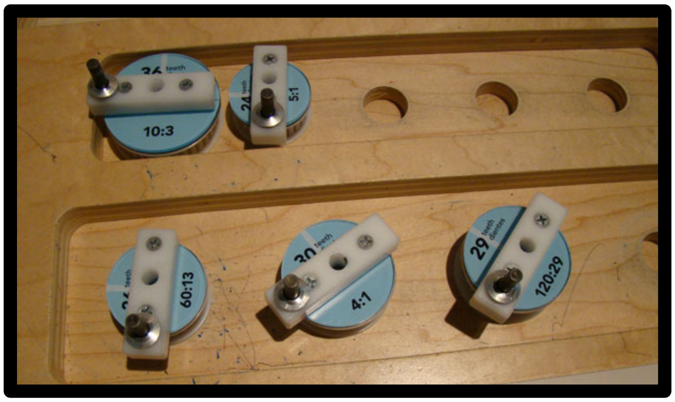


Figure 3.1: Most RRs ignored the numbers on the *Spirograph* gears, even when they recognized them as ratios. When they did use them, they tended to use them as identification labels, as in "disk 30."

This same family had been treating *Clicker* as a musical instrument from their first study visit, trying to set up interesting patterns of beats using the wheels. During the final study visit (which was one parent's third visit to this exhibit), the parent suddenly noticed that there were numbers on the wheels. After the group tried to figure out how they might use the numbers, the researcher gave the group some hints about lining up the arrows. This helped the group recognize that they could use ratios between wheel numbers and numbered positions on the rail to more quickly set up beat patterns. The adult said he was shocked that he didn't notice the numbers before this visit, and that he had actually found himself getting bored with this exhibit. But then, he explained:

> All of a sudden I was seeing all the numbers for the first time today and realizing that there was some relationship between the numbers. I never noticed them before. (C2-6)

This adult said that one thing he would change about the exhibits is adding hints to pay attention to the numbers to the instructions.

> Knowing those numbers is important, highlighting them somehow. Because I didn't notice the numbers there. Once you notice those, you try to create the relationship between the numbers. Then you have more control over what you're trying to do…because all this time [my 11 year old daughter] has been guessing. (C2-6)

Proportions are also involved in other exhibits, including for example, *Shadow Table*, *Big Shadows*, *Triangle*, and possibly *Balance*. It appeared that no other RRs figured these out, although it seemed that the same RR family might have noticed the proportion at *Triangle*, although they didn't call it that (and the word *proportion* does not appear in the label). The researcher's debrief described this experience.

> [Adult] said they were supposed to "line the [red squares] up with the long clear plastic thing." They said they noticed that the positions of the squares when they were lined up were "kind of doubles" 2, 4, 8, etc. (C3-3)

Figure 3.2: The RR noticed that the squares lined up in a "kind of doubles" pattern.

Related Math Concepts

In addition to visitors engaging with ratio and proportion, most of them also engaged with a number of additional math-related activities, for example, spatial reasoning, reading and using graphs, and balancing equations. Each of these is discussed briefly below.

<u>Spatial Reasoning</u>

Spatial reasoning was an important skill practiced by many respondents using many of the exhibits. It was prominent in particular at *Blocks*, both in completing the exhibit challenges, and also when respondents set up their own challenges like building structures of their own design (cubes, stair steps, pyramids, castles, and apartment buildings). Some respondents used *Blocks* in a manner similar to the popular game of Jenga (B3-3; D2-3; D2-6).

<u>Reading and Using Graphs</u>

Using graphs was an integral part of engaging with both *Rainbow* and *Sliders*, but it was interesting to note that many younger children learned to control the ways they drew the images even before they realized they were making graphs. For example, it was not unusual to see different respondents at *Rainbow* experimenting with moving at different speeds to adjust the slope, or at *Sliders* by moving the two knobs at different rates to make different shapes (D1-1). Many older children and adults indicated that they related the graphs at these exhibits to graphs they learned about at school and elsewhere.

<u>Balancing Equations</u>

Balancing equations appeared to happen primarily at the beam scale at *Balance*. While many RRs practiced hanging blocks on the scale, and in many cases were able to achieve balance, many of these interactions were intuitive in nature. Some other respondents used numbers to calculate ahead of time what they were going to do to get the beam in balance.

In summary then, the data indicated that many visitors to the *Math Moves!* exhibits engaged in all kinds of math activities and language including doubling, size comparison, balancing, matching, x/y axis, and more. But many of them did not readily recognize that what they were doing was math. As one staff respondent put it:

> My personal feeling is that every one of those exhibits is very much about math. What varies is the extent to which that's explicitly recognized. (SC4)

At least part of this may have to do with the traditional way society tends to view math, i.e. school math. But a bigger part of visitors not seeing the math in the exhibits appeared to be related to their notion that math involves manipulating numbers, and so many of the exhibits did not seem (to them) to focus to any great extent on manipulating numbers—even when they did.

3.2 How Did Respondents Engage With the Exhibits?

An important part of all visitor experiences is the extent to which and the way in which they engage with exhibits (Ansbacher, 2013; Ansbacher, Hein, McLean, Rounds, Spock, 2000). The topical framework (Appendix E – Topical Framework) identified four specific types of engagement: *physical*, i.e. what visitors did at the exhibits, such as stopping, jumping, hanging weights; *intellectual*, i.e. the thinking processes they used, such as performing mathematical operations, drawing comparisons, trying to figure things out; *social*, i.e. exploring things together with their visiting companions, such as working together to solve a problem, asking questions, directing attention; and *emotional*, i.e. the emotions that came to play, such as enjoyment, surprise, and frustration. The following section explores all four of these types of engagement.

Exhibits Stopped At

This study did not include a traditional tracking and timing study, but a log of exhibits visited by the RRs over all the study visits was kept. During the initial and final study visits, (when a researcher was on site) most RRs stopped at most of the exhibit units, at least in part because the researcher either suggested or strongly encouraged they do so. During the interim study visits, some groups made attempts to visit most of the exhibits, but in general, fewer exhibits were stopped at.

Although detailed data about length of stops at each exhibit were not recorded, not surprisingly, participating in this study appeared to increase the amount of time respondents spent at individual exhibits, and in the exhibition as a whole, especially during the initial and final study visits when they were being observed and videotaped. During the interim study visits, times at exhibits were less and more typical of times spent by casual visitors, although total time in the exhibition was still likely higher than would have been if they had not been part of the study. Exhibition visit times ranged from about 15 minutes during some of the interim visits, to over an hour, during some of the initial and final study visits. Because of the relative long dwell times at each exhibit, especially during the initial study visit, RRs had plenty of time to figure out how to work the exhibit mechanisms and then begin to explore what they might do there. During these times, they often discovered challenges that helped them begin to develop some of the basic math concepts that were built into the exhibit. Having the opportunity to make things, for example at *Spirograph*, *Blocks*, and *Drawing*, appeared to inspire some of the longest dwell times.

Excluding the exhibits that were only at Explora, it appeared that across all the museums the most visited exhibits, and the ones visited for the longest times, were *Rainbow*, *Blocks*, and *Spirograph*. *Clicker*, *Shadow Table*, *Balance*, and *Chairs* formed another cluster of popularity, a bit below the first group in frequency of visitation. The least visited exhibits were *Big Shadows*, *Shapes from Circles*, *Triangle*, and *Comment Board*. Because the layout and configurations of the four exhibitions were so different, it's impossible to tease apart the respective roles of popularity, location, and other factors. For example, *Spirograph* was visited fewer times than *Rainbow* and *Blocks*, but RRs often could not use it because other visitors were using it and tended to stay for long periods of time. And at SMM, *Big Shadows* and *Chairs* were sometimes not visited because the respondents didn't realize they were part of the *Math Moves!* exhibition.

What Visitors Did

Experiencing science museum exhibits is almost always a two-step process, although ideally these two steps happen almost simultaneously. The first step is figuring out what to do, while the second step is experiencing (or learning) something new or unfamiliar. Especially with many open-ended and more complex exhibits such as the *Math Moves!* exhibits, *developing fluency* is not limited to developing fluency with the content of the exhibit, but also with how it operates. It can even be argued that these skills are inseparable. For example, in order for a visitor to understand, develop an appreciation for, or evolve their fluency with the relationships between motion, speed, and distance at *Rainbow*, they need to learn how to operate the exhibit, i.e. to push the button to clear the graph, and to walk on the rainbow path while watching what happens on the monitor screen. With repeated use of *Rainbow* over time, this fluency with the mechanics of the exhibit increases, alongside fluency with using one's body to draw intentionally on the screen and to develop a more cognitive understanding of the interrelationships among direction of motion, speed, and distance.

As is typical with many science museum exhibits, initial use of the *Math Moves!* exhibits by respondents involved some members of the visiting group who jumped right in and began to immediately manipulate the exhibit—trying to figure what it does and how to use it—while at least one other member of the group (often an adult or older child) began by reading some labels. When monitor screens were included at an exhibit, most RRs focused their attention on what was happening on the screen (although children were often less apt to read the text on the screen).

During some early visits, many RRs also picked up the phones (that contained audio descriptions) at many exhibits, but often did not listen for very long, and they rarely used or talked about any of the information delivered that way. A notable exception to this was one adult respondent who – during the initial study visit – listened to every audio that was available, and kept trying to get the others in her group to listen to the audio before they did the exhibit. As she explained, this was the "correct" way to do an exhibit (D4-1).

Throughout all their study visits, most RRs eventually discovered many of the math tools available at each exhibit (e.g. the measuring devices, grids, numbered graphs axes, etc.) but it usually took at least a couple of visits for them to discover and use the available tools. Most groups used some math tools at some of the exhibits during each subsequent visit.

Over the course of the study, a few RR groups noticed and discussed some of the everyday life photo montages at the core exhibits but most RRs did not appear to notice or use them. And, over the course of all their study visits, most respondents tended to read increasingly fewer labels, and many tended to use more of the math tools.

Clicker

Blocks

Balance

Figure 3.3: A few RRs noticed and used the photo montages, but most ignored them.

How Respondents Used the Labels

Not surprisingly, many groups used the labels to facilitate their conversations. As is typical with many science museum exhibits, sometimes this was verbatim reading out-loud of some of the text on the label, and sometimes it was instant paraphrasing of the text into something more conversational or appropriate for someone in the group, e.g. replacing the word "large" with "big." At other times, the label text was read silently and carefully in its entirety, and then the exhibit was explained to others in the group in the respondents' own words.

During the initial study visits, most RR adults tended to use the labels by either reading them out-loud verbatim, or in some cases, paraphrasing them. They appeared to use them primarily to help support the teacher-like roles they had assumed. In some cases the adult naturally started a conversation about the exhibit (whether verbatim or paraphrased) whether the child was stuck or not, and sometimes the conversation was initiated directly in response to a child needing help. One RR group began each exhibit interaction by carefully and thoughtfully perusing the label, and trying to follow the directions as closely as they could. (B5-1)

As noted previously, many of the labels in the exhibits posed challenges to visitors. At some exhibits, these challenges were somewhat vague and thought-provoking rather than directive. Some examples were at *Chairs* where one label asked "How do you fit in this chair today?" and another challenged visitors to "Use your body and other tools to explore how the chairs differ in size."

At other exhibits, the challenge labels were more directive, such as at *Rainbow* which had drawings for visitors to try and reproduce, or at *Big Shadows* where labels encouraged visitors to "Make your shadow 8 squares high and 2 squares wide." and asked "If the shadow man stands on line 5, where must you stand to make your shadow the same height?".

By the time of the final study visits, all the labels appeared to get much less attention from most adults and children, although they were sometimes consulted to remind them of what challenges they could do there. When the labels included specific and directive challenges, and especially when there were multiple challenges as at *Rainbow* and *Big Shadows*, they were more likely to be used during the final study visit. But it appeared that for the most part, most RRs felt they had figured out how to use the exhibit, and what opportunities it afforded.

The Role of Exhibit Titles

Ideally an exhibit title cues the visitor about what the main goal of the exhibit is, and also what they are supposed to do (Perry, 2012, p. 151). Usually displayed prominently above the main label texts, many of the *Math Moves!* exhibit titles were written in a way intended to do just that. These titles included concepts and ideas that visitors could be thinking about as they used the exhibits—*comparing, scaling, ratios, fractions, half, whole, double, balance, gears*, and more.

The data indicated however, that few respondents found the titles useful in focusing their attention on the math concepts, or cuing them about what they were supposed to be doing. It could be that some of the titles used abstract language that was not particularly familiar. For example, instead of *Sensing Ratios* alternative (and more conversational) wording might have been something like *Hear, Draw, and Feel Ratios* or even simply *Hear a Ratio*.

For many respondents a word like *frequency* was confusing. Did it mean frequently, or did it relate to the tones as in sound frequencies? *Scaling Shapes* wasn't meaningful, but respondents talked readily about *doubling*.

Figure 3.4: Many titles were written in a way that was intended to focus visitor attention on the math concepts, but they did not appear to do so.

The Role of Exhibit Challenges

Research has repeatedly demonstrated that challenge is a critical component of informal learning experiences. Finding the right amount of challenge and carefully balancing it with feelings of confidence and competency without overwhelming or frustrating visitors can be a difficult proposition, especially considering the wide range of ages that visit museum exhibitions (Csikszentmihalyi & Hermanson, 1995; Perry, 2012).

Challenge tends to be incorporated into visitor experiences in three different ways: (a) the exhibit has natural affordances that subtly challenge visitors, for example, the large chair in *Chairs* naturally invites visitors to climb up onto it, and the bar at *Balance* encourages visitors to hang things from it; (b) the exhibit has verbal or graphical challenges incorporated into the labels, for example, text such as at *Rainbow* where the label encourages visitors to *Compare your rate of motion…..*, and another that depicts a line drawing of an elephant for a visitor to reproduce; and (c) visitors come up with their own challenges, e.g. at *Blocks* "Can I make a castle?" or "Can I make all the rabbits on the *Shadow Table* have the same size shadow?". In the best cases, visitors use the embedded challenges as jumping off points, and then develop their own challenges that are related to the exhibit goals.

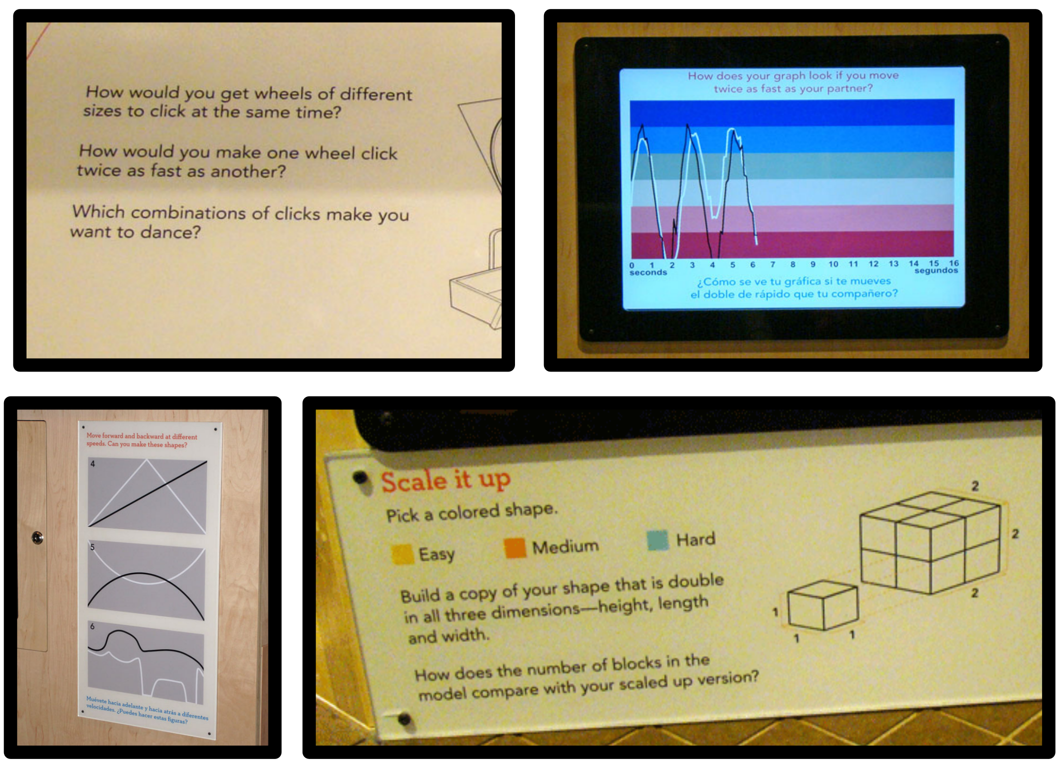


Figure 3.5: Many visitors used the exhibit challenges as a way to begin engaging with an exhibit.

In addition to the natural affordances of most of the *Math Moves!* exhibits, all of them included one or more (label) challenges for visitors to try (see Figure 3.5). Upon approaching an exhibit for the first time, many respondents used the exhibit challenges fairly extensively, often starting out with an initial challenge suggested by the exhibit, or by shaping their entire engagement around a series of exhibit challenges.

Oftentimes these exhibit challenges were used when one member of the group (usually an adult) read the label verbatim and then the group worked together to try to achieve the challenge. It was also not unusual for an adult to adapt the exhibit challenges by simplifying them, or leaving out the quantitative part, especially when they had a younger child with them. Some adults also made up their own challenges for their children, both younger and older ones, sometimes based on snippets of existing exhibit challenges but often not.

While many initial visits to the exhibits tending to rely heavily on the written exhibit challenges, during the later study visits there was evidence that most groups were still using some of the exhibit challenges, redoing or continuing to work on challenges they had done before. For instance, some RR groups tried the *Clicker* challenge early on (*How would you use wheels of different sizes to click at the same time?*), but were not successful. When they returned to the exhibit on a later visit, they continued to work on the challenge. Most of these RRs eventually succeeded—although some experienced significant frustration when, for example, the clicking drifted off beat over time.

Another example of how visitors were using the challenges to shape their visits was at the *Rainbow* exhibit. Most children tended to begin using *Rainbow* by engaging in a lot of exploring and playing around with little attention paid to the exhibit challenges. A few of these RRs eventually went on to attempt some of the exhibit challenges, but most needed encouragement and direction from their adult companions. If the adults intervened with suggested challenges too quickly, the children often resisted, insisting on continuing their exploration and free play. After repeated visits to the exhibit, some parents saw their children's skills at that exhibit improving and then pointed out the challenges to their children. The children then tried at least one challenge, carrying it through as best they could, but not always succeeding. By the final study visit however, when RRs stopped at *Rainbow*, most of them immediately selected a challenge picture they wanted to try, and were more successful than they had been during their earlier visits.

At many of the exhibits, many RRs eventually came up with their own exhibit challenges, especially after they had read the labels and either attempted the challenge and were successful, or, more frequently, attempted the challenge and were not successful. This happened most frequently at the more open-ended exhibits such as *Blocks* and *Shadow Table*. Many of these self-generated challenges were not related to the main goal or theme of the particular exhibit.

Taking a closer look at visitor engagements at *Blocks* for example, self-generated challenges tended to not focus directly on scaling, although many did tend to use a variety of math-related skills. Self-generated challenges at *Blocks* included building giant pyramids and a stair-step structure (A4-2, A4-3, A4-4, A4-5); constructing an apartment building like one her friend lived in (C3-2); building several simple cubic or rectangular structures – that were made more challenging because he incorporated the odd-shaped colored blocks along with the small brown cubes (A5-3); building a structure in order to remove blocks one at a time until it collapsed similar to the game Jenga (B3-3; D2-3; D2-6); and many examples of knocking structures down.

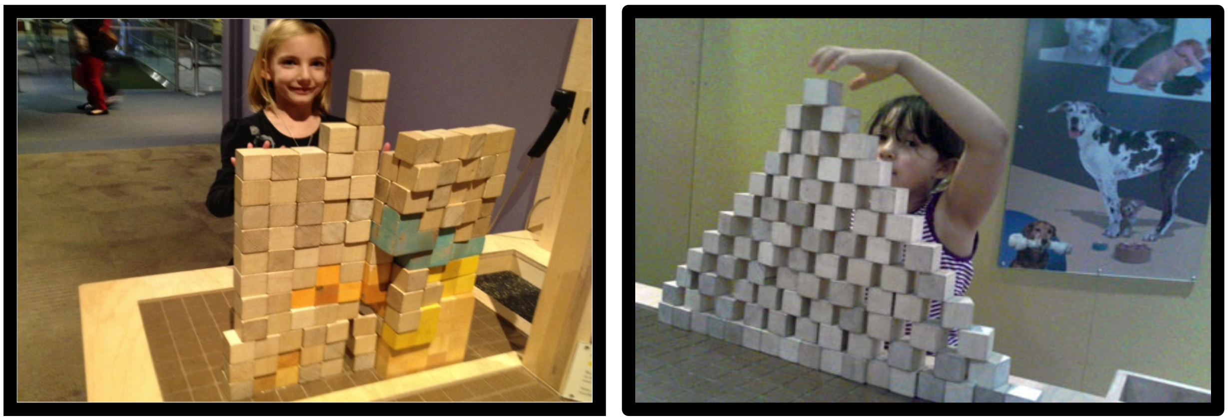


Figure 3.6: Some RRs came up with their own challenges at *Blocks*, many of which were not directly related to the exhibit's primary theme of scaling, but still were related to math-related skills such as spatial reasoning.

One bright and mathematically inclined respondent explained how he became bored at some exhibits and so came up with his own ways of interacting with them, some of them in ways that were only minimally related to ratio and proportion. Following is a detailed account of how he adapted the *Blocks* exhibit, and came up with his own challenge.

> [Child] reported liking to build forts and knock them down. He said that he and his brother "destroyed them". He said his father tried to teach him "about scaling figures but I kind of ignored it." He said (laughing), that he made his fort simply as something to destroy, low and wide—about 3 high and 15 wide. He said he brought a toy tank with him to the museum because he tends to eat lunch fast and he wanted to have something to play with while the others were finishing lunch at the café at the museum. The tank was small and he could put it in his pocket.
>
> At *Blocks*, he said he decided to use the tank….He and his brother built a small house around the tank then built 3 parallel walls in front of the house. His self-devised challenge was to destroy the 3 walls to get to the house. He said he and his brother used the pre-glued odd shaped blocks plus some single cubes to fill in the gaps, in order to make a rectangular prism for the tank's house. After building the 3 walls, they tried to [use] different methods to knock the walls down. First they used a block, "bowling" style but that didn't work. Next they tried (successfully) using a "train" style approach. They used a long string of blocks and pushed the "train" forward until it struck the bottom of each wall, which caused them to collapse. (D4-4)

In most of these self-designed challenges, there appeared to be minimal proportional reasoning and doubling going on. It could be argued however, that there was a fair amount of practice with spatial reasoning.

While most respondents who came up with their own challenges at *Blocks* tended not to focus on ratio and proportion, some others did come up with challenges that focused on (for example) doubling. For instance, in one group the mother and daughter raced each other to see who could duplicate the colored blocks more quickly (A1-4). The mom—who had described herself as someone who was not very good at math—appeared quite proud when she won the race (which turned out to be fairly frequently). A few other groups also came up with their own challenges focused on doubling (see Figures 3.7 and 3.8).

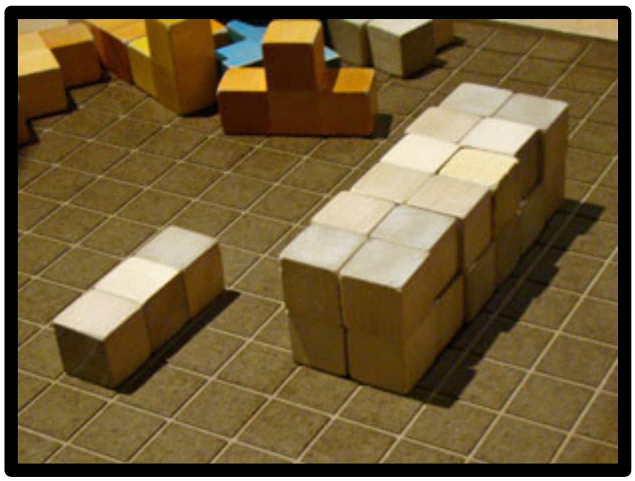


Figure 3.7: One RR child came up with his own challenge to try to double a structure that he had created on his own. (C5)

Figure 3.8: Another RR tried to double the size of the doubled-L structure shown in the video challenge, but she ran out of blocks before she finished. (C2)

The data indicated that for most of the recruited respondents, the challenge questions guided many of their experiences at the exhibits. It must be remembered however, that all of these respondents were highly motivated, and knew they were part of the study. They repeatedly let the researchers know both directly and indirectly that they took their roles seriously and wanted to perform well.

In looking at less motivated Casual Respondents (CRs)—i.e. those respondents who are more typical of most visitors to the exhibits—the data indicated that many visitors did not try to meet the exhibit challenges, especially when their parents were not present. Engagements in these cases included many more instances of such activities as moving objects around the *Shadow Table* without looking at or seeming to notice their shadows, making "woo woo" sounds at *Sliders* without watching their effects on the monitor, or running back and forth at *Rainbow* without trying to link body movements with what was happening on the screen.

While open-ended and free-spirited play and exploration is an important aspect of engaging with many science museum exhibits, it becomes a missed opportunity when visitors get stuck in this behavior, and don't see or take advantage of other opportunities. Much of this exploration and free-play could be enhanced if, for example, the visitor simply noticed and watched the shadows at *Shadow Table*, or the screen at *Sliders*.

While many CRs did not move beyond open exploration and play, some did use the challenges, for example, three teenagers at *Sliders* completing some challenge screens (including the 2:1 sloping line, and the circle) (CR4-20); two girls (about 9 and 11 years old) completing the third challenge picture at *Rainbow*, twice (after switching sides) (CR4-16); and some teens watching the video at *Blocks* and doubling a structure (CR4-20). In addition to using the existing exhibit challenges at the exhibits, there was also evidence that a few visitors made up their own challenges, such as making all the rabbits' shadows at *Shadow Table* the same size (CR4-16).

So, although challenges did not appear to dominate casual visitor behavior in the same way as they did recruited respondents' engagements, the data indicated that they were an important part of some CR experiences. In contrast however, the summative evaluation of another recent algebra-focused exhibition, *Design Zone*, found that the challenges set out in the exhibit labels in that exhibition were a large part of the casual visitor experience, with about 70% of respondents attempting one or more exhibit challenges, and almost 30% coming up with a challenge of their own (Garibay Group, 2013, p. 17). There are numerous possible reasons for this, and it would be an interesting topic to explore in more depth.

The Role of Archaeology

In this section we briefly discuss the role of archaeology on recruited respondents' experiences, with a focus on their initial and final study visits—those visit for which we have first-hand data. We are using the term *archaeology* to mean evidence of prior use of the exhibit—in other words, the way a previous user left an exhibit that has moveable parts, or examples of things that visitors can create. That could mean how visitors left the cutouts at *Shadow Table* and at *Big Shadows*, the weights on the *Balance* scales, the creations they left at *Blocks* or the *Comment Board*, or whether or not they left the *Spirograph* mechanisms set up and ready to go or disassembled.

Archaeology in this report, includes not only how visitors left an exhibit, and how that set up was then discovered and used by the next group of visitors, but also how museum staff and volunteers intentionally set up, or rearranged, an exhibit so that the next visitor would have a good starting point for their engagement.[9]

During the first study visits, archaeology appeared to play an important role at several exhibits for many RRs. For example, it was easier for first-time visitors to figure out *Clicker* if one or more wheels were already on the rail, and *Spirograph* if it was at least partially assembled; *Big Shadows* worked better if the cutouts (the shadow people) were left out where respondents could find them; and the papers others left behind at *Comment Board* served several purposes, including showing visitors what to do, providing inspiration, and pushing many visitors in new directions (because they wanted to do more than just copy other visitors' efforts).

In some cases, museum floor staff deliberately re-arranged the archaeology of an exhibit after a visitor left, to make it easier for the next visit, in a type of *intentional archaeology*. For example, adjusting the position of the weight on *Lever* so that the next visitor would be more apt to feel how heavy the weight was if they pushed down on the lever arm.

By the time RRs made their final study visits, archaeology left behind by previous visitors appeared to play a much smaller role. During these final visits, it appeared that whatever was left behind by the previous visitors often just got in the way of respondents' plans for their engagement, and thus objects were swept away so they could engage with the exhibit in the way they wanted. These findings imply that while archaeology can be an important starting point for first-time visitors, it probably has less influence on repeat visitors.

Group Interactions and Meaning-Making

Social interactions and joint meaning-making are important components of many science museum experiences (Borun & Dritsas, 1997; Diamond, 1986; Leinhardt, Crowley, & Knutson, 2002; McManus, 1987, 1988; Perry, 2012) and they were identified early on during this project as being one of the essential design characteristics. An important aspect of social engagements with the exhibits is the nature of the teaching-learning and joint meaning-making dynamic between the group members. With the RRs, it was interesting to note the wide range of adult/child interactions. This range occurred across the various RRs, but also varied from study visit to study visit, and even sometimes within a single visit.

A few parents took on a traditional teaching role, and said they wanted to make sure their children "learned." As described in the written debrief from one RR group:

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[9] Note that archaeology is just one aspect of Tatter's transactive approach to learning in museums, which influenced the set of design characteristics each exhibit strived for (Tatter, 2004, 2008).

> As they approached the exhibit, the parent said to the child "This is the one where you have to (reading the label) 'combining two straight movements to create curves.'" Then she pointed to the label and said, "That's a curve." (A4-1)

Another respondent described how she came up with questions for her daughter and had her answer the questions in a notebook. She said she had her do this so she could come up with the "right" answer. This is described in more detail below, in the section *Visitor Vignettes: Jessie & Estelle*.

Another RR adult repeatedly reinforced for her child that "the exhibits weren't just for playing but they were for learning." (D1-3) This parent also had the child work with paper and pencil at times, especially when it came to doing addition and subtraction while in the exhibition. The adult asked her child questions, and had her do math problems so she could come up with the "right" answer.

While a few adults stressed the learning over playing function of the exhibits, a few other adults took a slightly different but still traditional approach to working with their children, adopting a didactic and explanatory role (C1-1; C2-6).

In some situations, the teacher and student role seemed to flip, often within the same visit, depending on the particular exhibit and who had more experience.

> At some exhibits the mom gave advice and suggestions, at other exhibits (like *Rainbow*) the daughter seemed to have a better handle on what was going on, so she took the lead and gave her mom instructions. There were also quite a few competitions, with the mom often coming out ahead and the daughter getting only a little annoyed at that. (A1-6)

And in one RR group, the adult seemed generally less adept and confident at math so was a willing student while the children took on more of the teaching role, answering questions and explaining things (B1).

A different type of group interaction was when adults and children became co-learners, exploring the exhibits together, working collaboratively on challenges, jointly making sense of the exhibit. In some ways this type of learning was the most rewarding to watch, and appeared to be similarly satisfying for the respondents. In these situations, the roles of teacher and learner were often blurred and difficult to discern. The interactions were less about one person being a teacher, and one being a student. The experience was instead about the adult and child playing with and learning about math together, where the adult and child shaped each other's understandings (C2; B1; B2). Following is an example of this type of interaction at *Chairs*.

> [Adult] noticed they were to measure and compare the chairs and asked [Child] if he wanted to just make a wild estimate. [Child] read, "How many ways is the bigger chairs double the size of a smaller one? I'd say the size of the seat is pretty big." [Adult] asked, "Is it doubled?" [Child] said he didn't really know. [Adult] saw the measuring sticks and picked one up and headed for the medium chair and [Child] picked up a stick and followed her. [Adult] measured the chair seat and found the depth to be exactly one stick length. [Child] headed over to the biggest chair and said, "This one is two." [Adult] found the tape measure and said "We have another way to test it."
>
> …Meanwhile [Adult] used the tape and measured the depth of the medium chair. [Adult] joined [Child] at the large chair and together they measured it to be 36". [Adult] looked at [Child] and smiled and said that the other one was 18. [Child] said, "Wow. We can assume that the height is double, right?" At the same time [Adult] was saying she hadn't done width. She said that it would be the same length if it was a perfect square. [Child] said, "If it IS a perfect square." [Adult] said, "It kinda looks like it to me." And [Child] agreed. [Adult] said that this reminded her of Goldilocks and [Child] smiled. They measured and determined the seats were square and then measured the heights as 72 and 36. They determined the chairs were doubled in height, and the seat was doubled in width and length. "Completely double." (B1-1)

This type of side-by-side co-learning didn't take place frequently, but when it did it was fun to see. The respondents above were frequent visitors to the museum (although this was their first visit to *Math Moves!*) and they seemed very comfortable with learning in this manner in the museum, as well as with each other.

More typically, especially during the interim study visits when the researchers weren't on site as well as with many CR groups, parents tended to hang back while the children explored the exhibits pretty much on their own, choosing where they would stop, and how they would engage. In these situations, the adults were usually used as a resource, someone from whom to seek advice and help when the need arose, such as figuring out how to work an exhibit. In other situations with casual respondents, the child was older and the parent was someplace else in the museum.

In the situations when the adult was less involved (for whatever reason), most of the children did not have particularly mathematical experiences. This was especially true for casual respondents, but also true for many of the RRs (C3-5, C1-3).

As noted above, it was not unusual for adult roles to shift across visits, and even within visits. An example of the shifting adult roles comes from an internal data analysis document:

> One group had both parents along on the first visit. The dad was more mathematically inclined than the mom and, once he figured out the exhibits, he took on a somewhat didactic role in explaining them to his kids and his wife. The mom also supported her children's explorations of the exhibits, but in less didactic ways as they tried to figure things out together. The mom brought the kids for visit two, which seemed to be [primarily about using the exhibits haphazardly and without thinking or talking about math].The dad brought them for visit three, but he said he hung back and let the children use the exhibits mostly on their own. The final study visit was his third visit to *Math Moves!*. He sometimes stood back and watched at first, but usually alternated between acting like a co-explorer of the exhibit and a teacher who guided his child's explorations by asking questions and sometimes giving explanations. He also was a learner on this final visit, since I asked questions, gave hints, and gave brief explanations at most of the exhibits, and he often took the hints and ran with them. (Selinda Research Associates, 2013, p. 31)

It is tempting to attribute respondent groups' social interactions to being a function of their personalities, relationships, and family dynamics, and to a large extent it was. But a large part was also the extent to which the exhibit was able to support, guide, and facilitate rich and meaningful interactions.

When the adult was able to quickly glean enough from the exhibit design and labels to help support their child before the child lost interest, or to guide them in a way that directed their attention to the math in the exhibit, most children were able to experience the math and evolve their understandings and fluency in ways not possible when left to their own devices. But sometimes—in spite of their best intentions—adults inadvertently got in the way of their children's ability to play with the math and experience the richness of the exhibits. Sometimes this was because they took their teaching role too seriously; sometimes it was because they were not able to quickly grasp the mathematical concept underpinning the exhibits themselves; sometimes, even when they got the exhibit themselves, they didn't have the right words or ability to translate it into meaningful conversations with their children; and sometimes they did not have the tools or know-how to engage with their children in a meaningful way.

Affective Responses

The following section discusses three aspects of visitors' affective responses to *Math Moves!*, their responses to: (a) individual exhibits; (b) the overall exhibition; and (c) the math.

Affective Responses to the Individual Exhibits

In general, most respondents indicated that they enjoyed engaging with the exhibits. Even as early as the initial visit, it became clear that certain exhibits were going to be favorites among most of the children, especially *Spirograph* and *Rainbow*. However, most of the children's reactions to most of the exhibits seemed positive and enthusiastic and remained so throughout the study.

Some exhibits inspired surprise, amazement, or wonder the first time or two they were used, especially exhibits where unusual things happened, such as at *Spirograph*, *Chairs*, *Mirrors*, and *Magni-cam*. And some focus children seemed to respond particularly well to exhibit challenges, both the ones on the labels, and also the ones they set for themselves.

With exhibits where children made things, like *Blocks* and *Spirograph*, the RR children often made it clear that they were proud of what they accomplished, and they sought their parents' recognition. Most wanted to take home at least some of the drawings they produced (one girl said she wanted to display them in her new room). During interviews after the interim study visits, some adults reported that children had posted their drawings on bedroom walls and other places.

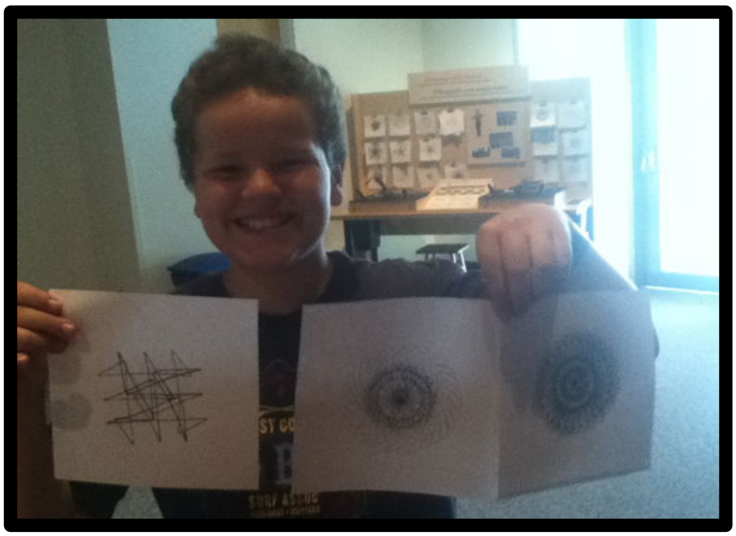

Figure 3.9: Some RRs were quite proud of things they created at the exhibits.

A few children became irritated with some of the parental "guidance" they received, saying they wanted to do something else, or engaging with the exhibits in a perfunctory way. Some RR children appeared to become bored with many of the exhibits, especially after the second or third study visit, preferring to spend their time deliberately *not* working on challenges at those exhibits, or instead simply not visiting those exhibits and focusing on their one or two favorite exhibits.

One young respondent in particular adamantly avoided *Balance* as much as she could, saying it was similar to something she had done in her classroom. She explained "It's hard….I don't really like doing this Mommy" (A1-1; A1-2; A1-3).

In spite of the overall enjoyment with many of the exhibits, some visitors became frustrated when an exhibit was overly confusing, or didn't work as it should. One example was the *Clicker* exhibit. During the initial study visit, many groups had trouble figuring out what to do at this exhibit. And once they figured out that they could try to get two wheels to click at the same time, many respondents had trouble accomplishing this goal.

Even when visitors were able to get two wheels to click at the same time briefly (usually by trial and error), the clicks often went out of sync quickly, apparently due to friction effects, a bumpy disk surface, or wobbly and unstable wooden wheel handles. Many respondents thought they had made a mistake and hadn't achieved the challenge, when in fact they had. One fourth grade RR repeatedly tried to get two wheels to click at the same time, and became frustrated when he felt wasn't able to, even when he actually was successful but then the clicking quickly went out of sync. The following excerpt from the debrief describes his experience.

> [His mom] said he did the matching challenge [at *Clicker*], but he couldn't get any of the wheels to match. She said she tried to help him, but he got really frustrated. She said he got mad at her and at the exhibit. When I asked [the focus child] which exhibit he did not like, he said the one where you "make both click at the same time." He said it was the hardest and was frustrating. I asked how they could make it less frustrating, and he said they could make lines that show where to put it so that they would go the same. (C5-3)

During the final study visit the researcher attempted to show this RR family how to use the numbers to quickly get the wheels to click at the same rate. The child said he had never noticed the numbers before, but even after they were pointed out to him, he declined to use them because he was focused on getting the wheels to click at the exact same time, and using the numbers/ratios only got them to click at the same rate (C5-6). Only two of the 16 RR groups figured out on their own how to use math/ratios to accomplish this goal, one during the third study visit, and one during the final visit. The group that discovered them earlier had forgotten they had done so by their final study visit (B1-6; C3-3; C3-6).

In spite of being a favorite exhibit for many RR children, another one that regularly elicited frustration among some visitors was the *Spirograph*. These visitors experienced frustration when they first tried to engage with the exhibit and it was not set up, or when it was time to change a gear for the first time. Some respondents also got frustrated at *Spirograph* when they could not figure out how to make it draw what they wanted. One five-year-old respondent wound up in tears when she realized she did not have any control over the outcome of the experience. (At one point she said she would have rather drawn her own picture.) The debrief from this visit describes the situation.

> The child is looking frustrated. She says, "I'm trying to make a flower but I can't." Her dad says "Keep going," and he then takes over turning the crank. The dad explains "You won't know the shape until you're all done."… The daughter puts her head down. The dad asks, "Why are you getting sad?" The daughter says, "I'm not a good drawer. I'm not a good drawer." The dad explains that the machine makes the drawing, not her. "The teeth and the turning makes a special shape. If you'll notice, it repeats, all the time."…"I still can't make a circle." The dad asks: "Why do you want to make a circle?" The daughter explains "Because [my sister] makes one."

> …The dad says that "The whole fun is kinda the mystery. What kind of shape is it going to draw? You can't change the shape it's going to draw by moving that arm one way or another….If you move it forward or backwards it's going to go over the same spot. Do you see that now?" The dad tells the child to turn it 20 times, and then they'll do something different. She does so very slowly, counting each turn. The dad says. "You see the pretty picture?" The dad shifted it to another post, then had his daughter do it 20 more times while he helps another child change a gear. [The daughter] is now in tears. She says she wanted it to "look like it did before." The dad talks to her in a soothing voice, then says she can do it any way she wants, but it's not going to make a circle. She turns it for awhile, and dad says it looks like it's making triangles. The dad asks if she wants to change it, and she says no. She decides to go do something else. (C1-1)

By the fourth or fifth study visit, this RR child appeared to have come to accept the fact that she did not have much control of the outcomes here, and began to enjoy whatever drawing was produced. Her dad explained that she was now more patient with turning the crank over and over again until the pattern had completely appeared. By the time of the final (sixth) study visit, this RR child identified the exhibit as one of her favorites.

Another respondent also expressed frustration when they weren't able to use the *Spirograph* to make what they wanted. This mom wanted to duplicate the really cool pictures clipped to the backboard of the exhibit, but she couldn't figure out how to do that.

> [The mom] notices all the other drawings that people made.… She asks if they can make the drawings posted on the board….[The dad] comes over and she points out the drawings on the board. She wants to know if you can decide what to make and [then make it]….[The mom] says she likes the shape she made, but she still seems to want to make the shapes on the board….The mom has figured out, "So whatever you put on here is what's making the shape?" She tries to change "this thing" [the gear] and has trouble. (C2-1)

A few respondents found the sound at *Sliders* to be particularly irritating and in some cases painful. This was usually alleviated once they found the volume control but sometimes it took a little while. *Shapes from Circles* frustrated some visitors because some of the disks were particularly difficult to turn. This resulted in movements that were not smooth, and which contaminated the experience and made it almost impossible for some visitors to feel the different shapes that were produced.

In many of these cases, after becoming initially frustrated with their experience, RRs eventually worked through their frustration and were able to engage with the exhibit—although this often was during a subsequent visit. With some recruited groups however, even though some family members had figured out an exhibit by the second visit and repeatedly tried to explain it, the remaining members of the group still couldn't figure it out. In these cases, the RRs became frustrated when they still couldn't understand the exhibit, even after they had engaged with the exhibit on two or three different visits. An example of this was a family group where the mom and older sister understood an exhibit, but the younger sister couldn't, despite several attempts by the mom and sister to explain it. (A4-2)

While the RR response to the overall exhibition remained generally positive overall, after the third visit enthusiasm began to drop off. By the end of study visit three, most focus children had at least a couple of exhibits they were no longer interested in returning to, including *Balance*, *Blocks*, *Chairs*, *Big Shadows*, and *Mirrors*. The primary reason given for this lack of interest was that the children felt they had done everything interesting at that exhibit—there was nothing more to attract their attention. It's important to note that the children had reached this judgment despite the fact that they had not yet figured out many of the math opportunities incorporated in these exhibits—at best they were just beginning to explore the ratios and proportions that were designed as part of those exhibit experiences.

At least a few of the exhibits that many focus children had become less interested in however were listed as favorites by other RR children, including *Blocks* and *Chairs*, and these children said they wanted to keep returning to these exhibits even after visit three or later. In some cases, even in spite of becoming bored with the exhibit, when an adult introduced a new challenge (for example when a dad challenged his daughter and son to build the tallest tower at *Blocks* interest was revived. (C2-4)

Affective Responses to the Overall Exhibition
As noted above, responses to the overall exhibition remained relatively positive throughout the study.

> I thought that what made it the most fun was how you were learning, like you were doing math and you didn't know you were doing it, and how it made you stay into it, like you didn't want to give up because it was fun and you're learning at the same time. (D2-2)

Many RRs indicated however that while overall they enjoyed the *Math Moves!* exhibits, they weren't their favorite ones in the museum. They explained that they were "less fun" than other favorite exhibits of theirs, but stressed that they weren't their least favorite. One family compared *Math Moves!* to other exhibits at Science Museum of Minnesota.

> [The RR child] said *Cell Lab* was the most fun [10 on a scale of 0-10], and *Minnesota History* was the least fun (0). The math exhibit as a whole was "a 4, maybe." [The child] said they gave it this score because "there's some stuff that's funner than this, and some that's not as fun as this." They said *Cell Lab* is fun because you can try different experiments and that they like science a lot. [The adult] points out that [their child] also gets to put on a lab coat and gloves and goggles, and mix things around and be more active. (The adult and child describe other things they do there.) [The adult] said that their 10 is *Wonder Years*…and 0 is *Minnesota History*, too. (The adult explains that it's more look and read instead of interact.) [The adult] says the math exhibits are a 5, a little higher, because they like math. They said a lot of the exhibits are nice, but there's a handful that just aren't that interesting to them. They said they don't think it's boring; there are just other exhibits that are more fun to do. (C3-6)

Another child compared *Math Moves!* to other exhibits in the museum.

> [The focus child said that] *Math Moves!* is not his favorite exhibit at the museum, and thought it actually might be his least favorite, although slightly better than the exhibit where you pour beads into different shapes to compare their relative volume….He said the first visit to *Math Moves!* was fun, but that the second time wasn't as much because he knew what to expect, and by the 3rd and 4th it got sort of boring. (D4-6)

And one RR dad explained "[There is] a lot of repetition….[The exhibition] is a little too small for me." (C2-3). One RR child explained that he would like to change the exhibits and "put some fun things in it." (A2-6)

Affective Responses to the Math in the Exhibits

This study included RRs with a wide range of knowledge about math as well as attitudes towards and comfort with math, so exploring how they perceived and felt about the math, was an important issue to explore.

Some respondents, especially those who were less comfortable with math, seemed to appreciate not being hit over the head with the math.

> I think the title of [the exhibition] makes you want to kind of shun away from it, because math doesn't excite me. But I think once you start the exhibit, it's not in your face that it's math. I mean, it just seems fun. And then you can see where the math comes out. But I don't think for a child that they see the math really….It says "*Math Moves!*" but it doesn't really strike me as math. I mean, it's subversive. It's subliminal in how it's teaching the math. I don't think the kids realize the math part of it. (C1-6)

For some respondents, finding the math in the exhibits made the exhibits more fun. One 12 year old RR and her father had been complaining about the exhibits being too easy and even boring. During the final study visit, the researcher began to point out ways that ratios could help them meet challenges at exhibits like *Clicker*, *Shadow Table*, and *Spirograph*. This seemed to revive these respondents' interest in the exhibits. The debrief from the final interview describes the situation.

> The respondent said that the exhibits weren't boring, because they weren't easy. It seemed like pointing out the numbers helped…The dad agreed, saying that the numbers at Shadows were a good thing. The daughter said that this time the numbers made it more fun. She said that trying to figure out the math made it fun, and that if it's really easy, it's not fun because she already knows it. But if it's harder it's more fun. (C2-6)

It should be noted in the above example that this respondent was not asking for a more difficult exhibit, but rather remarking that when the math was pointed out to her, she now had new ways to challenge herself. Nothing changed in the exhibit; the researcher merely directed her attention to things that were already there that she (and her dad) hadn't been able to find previously. By this time she was already bored with these exhibits because she thought she was done. Without the researcher's intervention and her participation in this study, it is unlikely she would have returned to these exhibits to engage any further.

Another young respondent described his enjoyment working with the math in the exhibits.

> If you compare it to other exhibits like the space exhibit—personally to me I find astronomy more interesting than math. But the exhibits in the math section are still fun which makes people want to go there and you kind of spend there longer because you have to think a lot about it. For the subject math…the exhibits kind of make the math fun because you have to bend around the exhibit and think beyond, instead of just what it tells you. (D4-3)

So it seems that both those respondents who enjoyed math and those who were less comfortable with it, found ways to enjoy the exhibits—the math-philes by having opportunities to dig more deeply into the math, and the math-phobes by feeling like they were not doing math at all.

3.3 Ways RRs Related the Exhibits to Their Own Lives

Overall, recruited respondents saw more connections to school math than to other aspects of everyday life. Some exhibits seemed more like school math, for example *Balance*, and the measuring at *Chairs*, and some—like *Rainbow* and *Clicker*—did not seem like school math at all, at least not at first. The photo montages of everyday applications of math concepts did not appear to be particularly effective at getting most recruited respondents thinking about math in everyday life.[10] The following section discusses two different ways RRs related the *Math Moves!* exhibits to their lives.

Relationships to School Math

A few RRs said they noticed things in the exhibits that reminded them of things they did in school.

> That chair measuring; I do that sometimes [at school]….We be measuring blocks, cubes, and all that stuff, plain shapes….it's circles, triangles. And these are real shapes: spheres, cube, cone. (D1-1)

And after seeing the equations label by the balance beam scale in the *Balance* exhibit, one parent reminded her son about the algebra he had done in a non-school summer math class. (A5-1)

A few RR parents also talked how what their children were doing at school influenced their exhibit experience. For example, some RR adults talked about how their children understood the graphs at *Sliders* and *Rainbow* better once the children had been studying different kinds of graphs at school (C5-3, C3-3, A5-3). Also, at the time of their initial study visit, some of the younger children had not yet studied multiplication, division, and fractions, but were beginning to learn about those subjects at the time of later study visits. Their parents talked about how, once they understood something about these basic math concepts, it was easier for their children to understand certain *Math Moves!* exhibits. One parent also talked about how she had her child practice multiplication tables as they went through the exhibits. (C3-4). And some RR children indicated that after they started learning X-Y coordinate systems in school, it helped them recognize the graphs at *Sliders* and *Rainbow* (B1-1; C1-5; D4-2; C5-2; C5-3).

Another family saw connections between doubling in the exhibits and learning doubling at school. This is described in the researcher's debrief.

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10 This study did not collect any data about how (or if) *casual* respondents related the exhibits to their own lives.

> The mom said that when you did something physical in the exhibit, and then you teach your child something on paper, you can refer back to what you did in the exhibit. They said they had done that, and I asked for examples….The dad talked about doing doubles for school math homework….“We can apply anything we do here almost on a basic level to what our children are doing at school. So it’s like a reinforcement. Coming here is another way to reinforce whatever it is we’re trying to teach them. It’s another way to look at it. You know, learning on paper is one thing, learning by hand is another. I think having a practical use of it will help any kid in math.” (C1-6)

During an interim study visit, another RR family talked about the relationship between *Math Moves!* and math the child was doing at school. This was described in the debrief from the follow up interview, which was describing their encounter with the *Comment Board* that asks: “How did you show double?”.

> [The child] told me that when she did the *Comment Board*: “I was like ‘doubles facts!’” [The mom] said she suggested to [her daughter] that she could maybe use something from one of the math worksheets she’s doing in school…that these worksheets had them doing a lot of coins where 2 dimes would equal 20 cents. [The daughter] agreed that “Yeah, that’s doubles.” The parent said that the daughter started drawing all sorts of options with money. The daughter drew an equation for me on Skype that was P+P=2P where the P was inside a circle to represent a penny. So it was 1 penny + 1 penny = 2 pennies.” (B2-3)

Due to the limited time of this the study, coupled with the fact that the study was not designed to go into classrooms, the data provided few insights into how respondents’ experiences with the *Math Moves!* exhibits contributed to learning math in school. Other studies indicate that sometimes exhibit experiences take a long time to percolate before they influence later learning in any obvious way (Perry, 2002).

Relationships to Daily Life

Some RRs indicated that they noticed and drew relationships between their everyday lives and the *Math Moves!* exhibits. Most of these tended to be in the form of simple reminders, or in some cases, analogies.

Two families noticed the everyday math photo montages and talked about hula hoops and how they related to how the mechanism worked at *Spirograph* (C2-1; C1-6), and another noticed and talked about the clock photo at *Clicker*, although the photo didn’t appear to relate it specifically to the exhibit (A4-1). Two different RR groups independently noticed and discussed the *Sliders* screen that related the stair-step pattern (which visitors were supposed to try and trace) to the music concepts of *unison* and *octave* (D4-2).

> The *Sliders* screen was on stair steps, which labels unison and octave difference. The first thing [Child] said was, "I play the violin which makes me notice these are an octave apart." She explained octaves on the piano….I asked how that relates to the screen and she showed me with the sliders that 3,4 was an octave different that 4,5….She went to the next screen which showed the diagonal lines of unison and octave difference and tried to trace those, noting that she thought 4,3 was Middle C. (D6-6)

One RR mom compared *Sliders* to a Theremin, and her daughter compared it first to an Etch-a-Sketch, and later to an "old synthesizer." (D6).

Some respondents made everyday-life connections on their own. Many RR adults called the exhibit with the official title of *Drawing with Gears* (i.e. *Spirograph*) a *spirograph* or sometimes a *spiralgraph*, since they remembered playing with the toy with that name as children. (C1-1; A4-1). One family compared the *Magni-cam* to a magnifying glass (A2-1), and another focus child compared the shapes seen on the *Magni-cam* screen to things they had seen (and/or imagined) before, including a stomach, the inside of an eye, a shark tooth, and Mammoth Cave (A1-6). One RR mom made a connection at *Shapes from Circles* when she asked her child "If you had a pencil in your hand, what shape would you be drawing right now?" (C5-1). One RR child said *Balance* reminded her of a video game she had played.

> There's this video game online that's like the balancing weights thing. It's like a tree branch and…it's sort of like one of those ancient Roman scales but there's a bird feeder on one side and a bird house on the other and you can fill the bird feeder up with seed however much you want but birds come down to your branch and you have to balance, and sometimes crystals form on one side. And baby birds too which are lighter and stuff you have to care about. (D6-1)

This same respondent also noted that *Clicker* reminded her of a clock. She went on to explain in some detail.

> There's a rotor inside [a clock] that turns the wheels and they go tick tick tick when they go around at a certain time. And the one hand is smaller, and it doesn't move as fast. And [at the exhibit] when you put the roller on the inside, it doesn't click as fast either. And the longest one on the very outside goes tick tick tick tick tick, and so does the one you put on the outside. So it just reminded me of a clock. (D6-1)

One young respondent talked about when they went to a playground recently, and played on a seesaw and got it to balance, just like in the *Balance* exhibit. She exclaimed "instead of blocks we did it with humans!" She talked about having a 3 year old on her side and two adults on the other, and that they got it to balance. She said they were pretending to be blocks, "but we didn't hang down, we hung up," which she realized was a good joke, "like hanging up!" (B2-3; B2-4)

One respondent talked about using *Shadow Table*, and related it to looking out into his backyard.

> [Child] noted it was frustrating that he couldn't get two bunnies to show up and be the same size because he found that the little one that was up close to the light, totally blocked the shadow of the big one that was farther away from the light. He said it was like his backyard. He noticed while looking out his window into the backyard that when the sun was at the front of the house, it made a big house shadow that went across the backyard. There is a tree in the backyard. He noticed that the house shadow totally obscured the shadow of the tree—just like the shadow of the bunny up close to the light obscured the shadow of what was farther away. He said the house and the tree at the exhibit reminded him of the house and tree at home. (D4-4)

3.4 Becoming More Fluent, Skilled, Appreciative Over Time

This study set out to examine visitors' experiences at the exhibits in *Math Moves!* with a particular interest in the ways in which and extent to which visitors evolved their understandings, appreciation, and fluency over time. This section will explore this issue in some detail, looking at various aspects of fluency, the role the exhibits played, and how respondent fluency evolved.

Knowledge Hierarchies

When describing ranges of understandings and engagements at museum exhibits, knowledge hierarchies are an established evaluation technique that has proven useful in many studies (Falk, Scott, Dierking, & Rennie, 2004; Gyllenhaal, Mackinney, & McClafferty, 1999; Mackinney & Bjork, 1999; McClafferty, 1996; McClafferty & Rennie, 1997; Perry, 1993, 2012; Rennie & McClafferty, 1996).

A knowledge hierarchy is a particularly useful technique for this study because it recognizes that all visitors to an exhibit are engaged in their own personal learning journeys, while at the same time, acknowledging that the journeys share some common characteristics. Visitors' learning journeys are personal in that some visitors have never heard or thought about the topic of the exhibit; some think about it in a limited or superficial way; and still others have varying degrees of sophistication and fluency. For example, some visitors to *Math Moves!* know about fractions; others are still learning to count; some understand gear ratios while others are unaware gears have teeth. All learning journeys however begin with a basic awareness, then some interest or curiosity, followed by some preliminary and then increasingly sophisticated understandings.[11]

Knowledge hierarchies by definition are theoretical frameworks derived from the integration of respondents' experiences and exhibit developers' (or the museum's) intentions. They are grounded in educational, motivational, and learning theories in such a way as to map virtually all visitors' experiences. Each subsequent level of a knowledge hierarchy subsumes the previous one. In other words, the knowledge hierarchy concept is based on the idea that a museum visitor cannot develop a sophisticated understanding of a main idea until they have a more basic, fundamental understanding of it. And they cannot have a basic understanding until they are curious or interested; and they can't become curious or interested until they are aware.

A knowledge hierarchy is not meant to imply that any one way of thinking is more or less important than another, or any "level" is better than another one. Rather, it's simply a technique that enables us to map visitors' unique journeys in a meaningful way. Some visitors to an exhibit

[11] Some have suggested that the term *trajectory* would be a better name to call what we are describing. While we may agree, the knowledge hierarchy is an established evaluation technique; re-naming it at this point is beyond the scope of this study. Whatever it is called, the underlying concept remains the same—a recognition that learning in a museum setting is unique to each individual, but that all visitors have in common journeys that begin with an awareness of the topic, an interest in that topic, and eventually an understanding of that topic that (ideally) increases in sophistication and fluency over time.

may need to repeatedly engage with an exhibit at a particular level, and they may never evolve in their understanding or fluency. For these visitors, repeated practice is the driving force, and this is a perfectly appropriate way of engaging. That said, it can also be argued that if all visitors to all exhibits never evolved their understandings or fluency, museums (and probably visitors too) would not be as satisfied with the experiences they were having as they would be if at least some visitors were increasing their fluency in meaningful ways at at least some of the exhibits.

Most knowledge hierarchies have five, six, or seven "levels," and are comprised of the same basic structure. *Level 3* is the basic core understanding or desired experience. Level 3 is the big idea, the main point, or the basic theme of the exhibit or exhibition. It's the exhibit's single main message or desired experience. It is however, not simply what the museum thinks the main idea is (although we often start there), but rather it's the intersection of the exhibit developer's intentions and the visitors' experiences.

Below Level 3 are three other levels. *Level 2* reflects less sophisticated, or sometimes inaccurate, understandings of the main idea. Visitors at Level 2 are often in the right direction of the main idea, or they have almost grasped the main idea, but they are not quite there yet. In some cases a Level 2 understanding is a misconception or alternative understanding. At an exhibit about seasons for example, the visitor may think seasons are caused primarily by the earth's distance from the sun. In other cases, Level 2 thinking is not necessarily wrong or inaccurate; it's just not quite what the museum was hoping for. At an exhibit about gold for example, some visitors may be thinking primarily about gold as jewelry, not recognizing its many other important uses.

Level 1 precedes Level 2 and implies a basic curiosity. It indicates even less understanding of or knowledge about the exhibit topic than Level 2. Level 1 is a very basic awareness or knowledge of the main topic, accompanied by curiosity to find out more. It is usually characterized by "I wonder." Level 1 is not a visitor who already understands the main idea and is curious to find out more. Rather it's the visitor who knows very little beyond a general awareness; but they are interested and have some questions. At an exhibit about Ötzi, the Iceman, they might say something like "Oh, I heard about him! I'm really interested in learning more."

Level 0 on the other hand is represented by a very limited knowledge about (or even an unawareness of) the topic, coupled with minimal (or a lack of) interest. Level 0 is often characterized by "Don't know; don't care." At an exhibit about the diversity of life in soil, a visitor might say something like "I don't know anything at all about that. I'm really not interested in dirt."

On the other side of Level 3 are Levels 4, 5, and sometimes 6 (although in this study, all hierarchies stopped at Level 5). Each of these levels varies greatly depending on the exhibit topic, but each reflects an increasing level of sophistication in visitors' thinking about the topic of the exhibit.

As noted previously, this study was particularly interested in visitor fluency. Fluency in this case does not refer to complete mastery, but rather a combination of knowledge and skill, and also feeling comfortable and at ease. As the findings from the study began to emerge, it quickly became apparent that there were at least five different fluencies that were important in the *Math Moves!* exhibits and that each fluency could be described by its own hierarchy. These included fluency with: (a) engaging with and using the exhibit; (b) using basic math skills (such as counting, adding, and measuring); (c) qualitative and kinesthetic understandings of ratio and proportion; (d) understanding quantitative relationships; and (e) using numbers at the exhibit. In addition to these five fluencies, two hierarchies emerged dealing with visitors' (f) perceptions and understandings of the main point of the exhibit (or about the exhibition as a whole); and (g) appreciation for math, ratio, and proportion as part of everyday life.

While recognizing that it is somewhat artificial to pull apart these different types of fluency that are, in practice, hard to separate, we found that it was useful to tease them apart because they often evolved at different rates and were evidenced in very different ways at each exhibit.

The seven hierarchies identified above are each presented and fleshed out below. While reading the following section, it is important to keep in mind that a knowledge hierarchy is a theoretical construct. While deeply grounded in the respondent data, knowledge hierarchies also share a common (pre-determined) structure, a structure that showcases increasing sophistication and fluency. However, whereas the basic structure of the knowledge hierarchies below is pre-determined, the knowledge hierarchies themselves were not pre-determined; they emerged from the data.

In other words, all of the following hierarchies were developed by listening to and talking with exhibit developers and members of the project team, *and* by observing and talking with visitor respondents. As findings emerged from the data, preliminary hierarchies were developed, and then tested against subsequent findings, revised, tested again, etc. This was an on-going, mutually-shaping process that evolved during the course of the study.

It's also important to note that the development of these hierarchies was not dependent on identifying respondents representing each of the levels. In other words, these hierarchies represent all *visitors* to the exhibitions, not necessarily all *respondents* in the study. For example, in developing a hierarchy we recognize that all learning journeys start with a lack of awareness and no interest (Level 0). Even though we may not talk with a respondent at Level 0, given unlimited resources we are highly confident that there are at least a few visitors out there who have no awareness or interest in the exhibit topic.

The hierarchies below are presented in no particular order. Each level begins by using first person narrative, as if a visitor is talking. These are not direct quotes, nor are they representative of any particular visitor; rather they are simply an attempt to paint a picture. It's also important to note that some of these hierarchies were more of a focus at some exhibits, and less of a focus at others.

3.4a Engaging with and Using the Exhibit

One of the first challenges visitors to an interactive science museum exhibit encounter is the physical operation of the exhibit. What am I supposed to do? How do I engage in a meaningful way? Beyond ensuring that visitors would be able to successfully operate the *Math Moves!* exhibits, the exhibits were designed to be open-ended and exploratory, enabling visitors to come up with their own challenges, problems to solve, and personal goals. Inherent in the design was the opportunity for visitors to engage with the concepts of ratio and proportion in any number of unique and exploratory ways.

0 I don't know how to operate the exhibit, and I am not really interested in trying to figure out how to use it as intended. I may not even really be aware of the exhibit; it's merely backdrop to something else I am doing.

Respondents at Level 0 included those respondents throwing blocks at *Blocks*, or playing with the cutouts at *Shadow Table* without looking at the shadows they produced. Some of the most frequently seen Level 0 engagements took place at *Rainbow* when children would race back and forth many times, racing each other—but not paying any attention to the screen; at *Sliders* when respondents would push the levers up to the loudest point and then leave them; or at *Balance* using the cubes to make a necklace.

Note that Level 0 engagement is not necessarily undesirable or inherently negative. In fact many respondents needed to start here before they could engage with the exhibit in any other way. However, if all respondents remained at Level 0 during all six study visits at all exhibits, we would argue that their experiences were not as rich as they could have been.

1 I don't know what this exhibit is about, but I wonder what happens when I do something. I wonder what I can do at this exhibit. I may start messing around with the exhibit, just to see what it will do.

Respondents at Level 1 were trying to figure out what the exhibit was supposed to do and be about. This was often by trial and error, for example when a respondent would "scribble" on the screen at *Sliders*. Level 1 sometimes required assistance from the adults in the group. At many exhibits (like *Blocks*, *Chairs*, and *Shadow Table*) figuring out the exhibit was fairly straightforward and the group quickly became more fluent. At other exhibits (like *Clicker*, and *Spirograph* when it wasn't set up), it was more complicated and took longer. It was not unusual for most RRs to return to a Level 1 fluency for a short time when they re-visited an exhibit after being away for some months.

And some RRs never increased their fluency past Level 1 at a few of the exhibits (like *Clicker*) because they were too complex for them to figure out.

2 I think I know how to use the exhibit, or I'm just starting to figure it out. I may not be using the exhibit entirely as intended, or perhaps I am using it in a superficial or

limited way. For example, I may be using the knobs at *Sliders* to make higher or lower pitched sounds, noticing that there is a relationship between what I am doing and what's on the screen, but not aware of what that relationship is. Or I may be using the cubes at *Balance* to count or add.

In some cases I may not be able to successfully use the exhibit to explore ratio and proportion because it's broken, or because I can't figure out what to do.

Respondents at Level 2 began to develop preliminary ideas about the exhibit and how to use it, but their explorations were incomplete, naïve, or inaccurate. Oftentimes, respondent Level 2 explorations included attempting some challenges, either those suggested by the exhibit labels, or in some cases by adults or other visiting companions, but they weren't able to successfully complete the challenges.

3 I get the basic gist of how to use the exhibit. I'm using the exhibit to explore basic mathematical relationships, even though I may not be aware that that's what I'm doing and even though I may not be able to articulate it. I may still not be aware of all that the exhibit has to offer, but I can use it to engage with ratio and proportion successfully. At *Chairs* I might use the measuring tape or squares to compare the height of the small chair with the middle chair. At *Sliders* I may use the knobs alternatively to systematically trace the stair step. At *Rainbow* I know that I need to hit the Start button to clear the screen; I know that where I stand and how I move dictates the line that will be drawn.

Respondents at Level 3 were able to quickly and successfully use the exhibit to explore basic relationships between objects, and engage with ratio and proportion. After they had figured out the exhibit, when they returned during subsequent visits, many RR children continued to explore them in the same way, not increasing their fluency very much for the remainder of their study visits. They repeatedly achieved success with the basic operations, but did not recognize that there were additional ways they might be able to use the exhibit to come up with their own challenges, or explore ratio and proportion in new ways. For example at *Blocks*, after doubling the basic shape during the first and second study visits, some respondents became bored because they felt they had "done" the exhibit. Or at *Chairs* once they had figured out that each chair was double the size of the smaller one, they figured they were done, and didn't want to use the exhibit any more.

4 I know how to use the exhibit quickly and efficiently. I'm engaging with it in more sophisticated or complex ways. I am able to successfully use the exhibit in more than one way to explore ratio, proportion, and related math concepts (even though I may not use those words to explain what I am doing). At *Rainbow* I can plan and draw many different shapes by adjusting my movement. At *Blocks* I can double a variety of structures. At *Shadow Table* I quickly and fluidly move the figures to double and triple the size of the shadows.

5 I easily and fluidly take advantage of many different opportunities that the exhibit has to offer. I explore ratio, proportion, and related math concepts seamlessly, efficiently, and fluidly, in a variety of creative and increasingly sophisticated ways. I set and achieve new challenges for myself using the opportunities provided by the exhibits, even though I may or may not be able to articulate that that's what I am doing. For example, I might use *Rainbow* to try to figure out that I can't draw a circle because I can't go backwards in time.

All respondents needed to figure out at least some basic ways of operating the exhibits in order to engage with them. At the beginning of the study, it was sometimes bumpy and even frustrating, but by the second or third visit, most RRs had figured out the basic operation of many of the exhibits. By the final study visit, most RRs approached most of the exhibits with a certain familiarity and ease, and were able to use them comfortably and mostly successfully.

Fluency in the mechanics and operation of the exhibits was very different for each one. For instance, at *Rainbow* visitors had to find the start button and push it before they could make a graph. At *Clicker* they had to figure out that the wheels should be set on the large disk in a certain way. At *Spirograph*, when the mechanism was disassembled, they had to figure out how to assemble it, put in a piece of paper, and perhaps change a gear. It was not unusual for respondents at certain exhibits such as *Rainbow* and *Big Shadows* to throw themselves wildly into the experience at first before eventually settling down and trying to make sense of what was happening.

The findings clearly indicated that the sixteen families all evolved their fluency in the mechanical operations of most exhibits. By the final study visits, all were jumping right in, and easily demonstrating that they knew how to operate the exhibits. They seemed to know exactly what they wanted to do and how to do it, and they felt confident in their ability to show the exhibits and how to engage with them to others. While all RRs became more fluent over time at most of the exhibits, with three months or more between some visits, there was also some forgetting taking place, and during later visits the first 20-30 seconds at an exhibit was often devoted to figuring out what had been forgotten.

Respondents were able to evolve higher levels of fluency more easily at some exhibits than at others. For example, achieving high levels of fluency at *Clicker* was difficult due to both the tendency of the wheels to stick, and also RRs mis-reading the numbers on the paddles (see the section below "3.6 When Things Go Awry"). For many RRs, fluency at *Rainbow* was inhibited because of the graph lines dropping out when they stepped off the edge of the path. And at *Spirograph*—although most respondents became quite fluent over time creating drawings and even mastering switching gears—few increased their fluency to where they were able to successfully use the exhibit to explore ratio and proportion. At this exhibit, in order to successfully explore ratio and proportion, you had to understand that each gear combination made a different pattern, that the pattern changed if you put the other end of the arm on a different post, and that you had to turn the crank lots of times to make the full design.

At some exhibits, such as at *Spirograph*, many RRs found it satisfying enough to simply continue to draw pictures (never increasing in fluency beyond Level 2), or at *Blocks* to build interesting sculptures without engaging with ratio and proportion, oftentimes even after successfully doubling a structure. At others, such as at *Clicker*, some RRs became frustrated because they were unable to be successful, for example when they didn't think they could make the wheels click at the same time (even though in some cases they actually did). In these situations, the respondents often chose not to engage with the exhibit during any of their subsequent visits, in effect reverting to engagements that looked more like Level 0 ways of engaging.

And at some exhibits, such as at *Rainbow*, and to some extent *Clicker*, a few RRs got caught up in a particular challenge that side-tracked them from spending time exploring ratios and proportions, for example the RR at *Rainbow* who repeatedly tried to get the line as smooth as in the drawing, or other respondents who spent large amounts of time trying to figure out (usually incorrectly) how to achieve a challenge without the lines dropping out. Also at *Rainbow*, some respondents spent a large amount of time trying to figure out how it worked: it was by your weight; you have to shuffle; it's about your arms; there's a camera; if you lean forward without moving your feet you can make the line move; etc.

In a few of the exhibits, fluency with operating the mechanics of the exhibit also sometimes evolved as the child physically developed, for example, as their arms grew longer, some children became more fluent in operating *Sliders* by themselves (rather than using one slider as their adult used the other). Regardless of physical limitations, most younger RRs in particular needed significant amounts of assistance from their accompanying adult in order to successfully operate the exhibit and achieve a basic Level 3 fluency.

Not surprisingly, recruited respondents achieved more sophisticated levels of fluency much more often than casual respondents. This was partially because their attention was more focused, and partly because we followed them through multiple visits.

We found that respondents' levels of fluency in operating different exhibits was fundamental to what they were able to take away from their experience, as well as influential in the fluency they were able to achieve in the other hierarchies. By the later study visits, many RR adults commented on how much more skilled their children seemed to be at various exhibits—especially the graph-making exhibits such as *Sliders* and *Rainbow*—and during the final study visits many young respondents were showing off their skills to the researchers. This type of operational fluency improved over time at most exhibits, for almost all respondents. Few children performed flawlessly every time, but most respondents were much more fluent during their final study visits than during their initial ones.

3.4b Using Basic Math Skills

The *Math Moves!* exhibits required varying degrees of knowledge and skill with math. This hierarchy explores the range of basic skills visitors had including counting, measuring, adding, subtracting, multiplying, dividing, and reading a graph.

0	I don't know how to measure, count, or employ other math skills.
1	I'm curious. I wonder how to measure, count, and use basic math skills.
2	I have some basic math skills, but my fluency with these skills is limited. For example, I may be able to count, but only up to a certain number. I may be able to add and subtract, but only with small numbers, or I tend to make lots of mistakes. I may understand the gist of how to measure something, but I make a lot of mistakes when I try to measure, or I don't feel comfortable with measuring.
3	I have a solid set of basic math skills including counting, adding, subtracting, and measuring. I'm usually, if not always, successful performing these basic skills.
4	Counting, addition, subtraction, and measuring are second nature to me. I perform them intuitively and fluidly, and rarely make mistakes. I also am able to successfully multiply and divide, and I know how to read a simple graph.
5	I have a sophisticated understanding of math skills beyond the basics listed above. I'm able to easily and fluidly use a variety of mathematical operations in a variety of ways. I understand how to read and interpret different kinds of graphs. I understand sophisticated ideas about ratio and proportion.

Once the basic operational mechanics were worked out, at many of the exhibits respondents carried out mathematical processes or tasks that they could get better at with practice. At some exhibits, the visitor tasks were overtly mathematical—like using a tape measure at *Chairs*, or counting and multiplying the weights to get the beam to balance at *Balance*, or reading a graph at *Sliders* and *Rainbow*. At others, they were more implicitly mathematical, such as using one's body as a tool for drawing at *Rainbow* (on a large scale), and drawing at *Sliders* (on a smaller scale). Most respondents gained at least some fluency with the various types of mathematical skills as they practiced over time.

The youngest RR children came to the study with some counting, measurement, and addition/subtraction skills, but many respondents had not yet studied multiplication, division, fractions, and such. Even ideas like *double* and *half* seemed new to some of them. Since most parents seemed to be aware of what their children didn't know, they sometimes simplified what they said about the exhibits to their young children. Some of the older children were still learning about fractions, graphs, and so forth, but they generally had an easier time with basic math skills and operations that involved multiplication and division. Most children's fluency with math skills improved during the year-and-a-half of the study, partly because they were studying math at school or picking up some aspects of math on their own at home, and partly because of their exhibit experiences in *Math Moves!*.

The data indicated that most respondents developed their math skills elsewhere (like school), and brought them with them to the exhibit. Over the multiple visits, younger respondents in particular improved their math skills at school or home, and then were able to use the exhibits in more sophisticated ways. For example, during the course of the study respondents may have practiced measuring at school or home, and this then enabled them to engage with greater fluency at *Chairs*. In other situations, learning a *new* skill at school such as multiplication, or how to read a graph enabled them to develop greater fluency with exhibits such as *Balance* or *Sliders*.

Because we weren't able to follow respondents home or into the classroom we don't know the extent to which respondents' experiences with the exhibits influenced or contributed to skills at school or at home. We did explore this issue with most respondents during the interviews, but found only a few examples of *Math Moves!* exhibits contributing to respondents' engagement with or understanding of school math (see description and quote below). This could be because (as will be discussed in more detail in the section *Main Point of the Exhibit*) most respondents did not focus on concepts related to ratio and proportion when they used the exhibits. It could also be that any such effects will take a long time to develop, as has been demonstrated in other studies on the long-term effects of experiences in museums (Perry, 2002; Spock, Perry, & Lewis, 1997).

One RR family group in particular described how engaging with the *Math Moves!* exhibits did influence how they approached school math, and helping them to feel more confident. This group was not particularly positive about math prior to *Math Moves!*. The child indicated that he noticed (and appreciated) how the exhibits were different from school math. And the adult explained that she now felt less intimidated by math.

> [The focus child] talked about how now he likes math that's hands on. He said he now knows that there are fun ways you can do math….[His adult] said that her feelings about math have definitely changed. "It's less daunting to me, especially with him getting into middle school now and having more challenging math assignments that he's coming home with. Now I can think about it in a little different way of trying to help him with it in a way that makes sense to me, and hopefully it will make sense to him." [The adult] said that sometimes they get confused by the [math] assignments or by how the teacher talks about them. [The adult] talked about how she used the balance beam as an analogy to explain conversions of weight measures (ounces to pounds), using her hands to simulate the scale balancing. "Just to think of it a little bit differently, that might help [the focus child] understand it better. I think [previously] he looked at it as just memorizing….But if you just think of it a little bit bigger, that might help him do better." (C5-6)

With adult visitors, the data indicated that many who did not use math much in their daily lives had to be reminded of how to apply the math skills used in the exhibits (and sometimes they remembered wrong). A few had limited knowledge of some of the basic skills, resulting in them making simple addition or measuring errors. This inhibited their (and their children's) ability to develop greater fluency, and also limited their ability to guide and engage with their children in meaningful ways.

In a few cases we could see young respondents' skills with math evolving as they used the exhibits, for example recognizing the richness and complexity of doubling at *Blocks* or using the measuring devices at *Chairs*.

A few adults purposefully used the exhibits to help their children practice math skills they were learning at school. For example, in one RR group, the child learned about multiplication at school between the second and third study visits. At subsequent visits, the adult looked for ways to practice multiplication at *Blocks* and at *Clicker*, among others. "Any chance to multiply is good." (C3-3; C3-4)

Making mistakes seems inevitable when people do math, and *Math Moves!* was no exception to this rule. For instance, especially during the initial study visits, some RR children made mistakes taking measurements at *Chairs*. These measurement errors prevented them from seeing that each chair was exactly double the size of the next smaller one. Some children also had trouble adding or multiplying numbers larger than, say, 20. Some respondents added when they should have multiplied.

Misreading numbers was another type of error that had important implications for understanding certain exhibits. For instance, one parent misread 1:2 on the ratio scale label as "1 point 2," which prevented them from understanding what can be done with this scale. Some respondents misinterpreted the numbers on the *Clicker* wheels as fractions. The numbers are really wheel diameters in inches. One respondent repeatedly read the number "72" as "27" when measuring at *Chairs*. When respondents made these sorts of misinterpretations, it was virtually impossible for them to discover the role that proportions play in meeting the label challenges, and often led to frustration, and feelings of inadequacy.

Overall, many respondents increased their fluency in basic math skills during the course of this study. Most of this increase however appeared to be a function of what they learned in school rather than at the *Math Moves!* exhibits. There was little evidence that most adult respondents increased fluency with their basic math skills as a result of engaging with the exhibits.

3.4c Qualitative and Kinesthetic Understandings of Ratio & Proportion

This hierarchy explores respondents' non-numerical understandings of ratio and proportion. It includes intuitive understandings, as well as those that are often referred to as embodied math, or bodily kinesthetic learning (Gyllenhaal, 2006; Nemirovsky & Gyllenhaal, 2006; Wright & Parkes, 2015).

0 I'm not thinking about relationships between objects, and I don't care about comparing things at this exhibit.

1 I wonder about basic relationships between things I see in this exhibit. Is one bigger or smaller, faster or slower, lighter or heavier?

2 I know that there are relationships between things in the exhibit, but my understanding is simple. It may be somewhat naïve and unsophisticated. For example, I may recognize that some things in the exhibit are bigger or smaller, faster or slower. I may move my body in certain ways but I don't yet fully understand how my movements make things larger or smaller.

For example, respondents at *Shadow Table* and *Chairs* talked about objects being bigger or smaller; at *Pulleys* about being faster, slower, or the same speed; and at *Heights* about being taller, shorter, or about the same size.

3 I get the basic idea that there are relationships between things in this exhibit, and that some things are bigger/smaller, faster/slower, lighter/heavier. I can move things (including myself) to make them bigger/smaller, faster/slower, heavier/lighter. I see that different things (objects, motion, sound, etc.) have different kinds of relationships.

Respondents at Level 3 noticed patterns such as the speed of clicks increasing or decreasing when the wheels moved closer or further away from center at *Clicker*, or that shadows would get larger or smaller when you moved the objects closer or further away from the light source at *Shadow Table*. At *Rainbow* it was recognizing that if you want to make a steeper line you move faster, and if you want to make a horizontal line, you keep still, regardless whether you can articulate this or not.

4 I have a solid intuitive understanding about relationships at this exhibit, regardless of my ability to articulate those understandings. I have a good understanding of the strength of relationships between things in the exhibit (objects, motion, sounds, individuals). I understand that when I move in this way, or arrange parts of the exhibit in that way, it can change the relationship to other things in the exhibit in a particular way.

Level 4 included (for example) respondents at *Rainbow* fluently moving their bodies to make a pattern on the screen by coordinating their direction and rate of movement while watching the screen, and constantly comparing the image that appeared on the screen with the shape on the label. It included visitors who fluidly executed the stair step at *Sliders*, or intuitively placed objects at *Shadow Table* precisely where they needed to be to create the shadows they wanted.

5 I have a strong and clear understanding of the relationships between objects, sounds, and movement in this exhibit. I move fluently and fluidly to explore a variety of different types of mathematical relationships between objects in the exhibit. I'm able to hypothesize about and successfully predict the relationship between different objects, including adjusting my movement, and directing another person's movement. I can fluidly accomplish whatever ratio and proportion goal I set for my exhibit use.

Many respondents used the exhibits in a variety of qualitative and kinesthetic ways, and many of these engagements slowly evolved in sophistication and fluency over time. At the same time, it was not unusual for many respondents to develop a primary way of engaging with an exhibit, and maintain that type of engagement during their subsequent visits.

Engagements demonstrating increasing qualitative and kinesthetic fluency were particularly noticeable at whole body exhibits like *Rainbow* and *Big Shadows* and less noticeable at exhibits that emphasized more explicit and quantitative math skills such as *Balance* and *Blocks*. For example, some RRs became quite skilled at using their bodies to draw the graphs at *Rainbow* and at *Sliders*, in some cases increasing their fluency to a Level 4 or even 5. These respondents appeared to develop their intuitive sense of how to get the lines on the graph go precisely where they wanted them to, even though they didn't use numbers and were unable to articulate why they could make it do what they wanted it to (C5-6). Fluency also increased over time at *Shadow Table* where respondents developed intuitive understandings of how to manipulate the figures to create the shadows they wanted.

And at *Spirograph*, even though they didn't evolve their kinesthetic fluency to any significant degree, some RRs continued to make new and interesting comparisons among the drawings they made with different gears and combinations, indicating a slight increase in fluency within Level 2.

In general, most respondents increased their kinesthetic fluency and qualitative understandings of ratio and proportion at most of the exhibits, although they were often unable to articulate what they were doing or why. This was especially evident in the exhibits that involved larger body movements such as *Rainbow*, *Sliders*, and *Big Shadows*. But it also happened to some extent at smaller exhibits like *Shadow Table* and *Balance*. Some respondents developed a primary way of engaging qualitatively—such as squeezing onto the smallest chair at *Chairs*, and then struggling up onto the largest chair—and remained with this type of engagement over multiple visits.

3.4d Understanding Quantitative Relationships

This hierarchy explores visitors' understanding of quantitative relationships between objects in the exhibit, regardless of their use of numbers. While visitors may have used numbers, the important distinction in this hierarchy is the focus on the quantitative *relationships*, not the numerical tools—the focus of the next hierarchy.

0 I'm not thinking about ratio and proportion, making comparisons, or the relationship between objects in this exhibit.

1 I wonder about basic relationships between things I see in this exhibit. Is one bigger or smaller, faster or slower, lighter or heavier?

2 I recognize that some of the objects in the exhibit have a simple quantitative relationship but my understanding is naïve. For example, I might notice that the middle chair is bigger than the small chair at *Chairs*, but I haven't figured out how much bigger. Or my understanding may be incomplete or inaccurate. For example, I might double the structure at *Blocks* along only one dimension.

3 I get the basic gist of the primary quantitative relationships at the exhibit. For example I might know that the middle chair is twice as tall as the smaller chair at *Chairs*. Or I might know to put equal quantities of blocks on either side on the scale at *Balance* (a 1:1 ratio). At *Shadow Table* I might notice that one shadow is twice as tall as another one. At *Blocks* I might double a simple structure.

Many Level 3 engagements on this hierarchy focused on simple doubling, such as at *Blocks*, *Chairs*, and *Big Shadows*. Most RR adults and older children readily grasped this type of doubling, but it was more difficult for those who had not yet studied multiplication at school.

4 I understand some of the more sophisticated quantitative relationships among objects at the exhibit. For example, I might know that not only can I balance the scale at *Balance* by having equal numbers on both side, but I also recognize that I can manipulate the distance from the center as an additional variable. At *Blocks* I am able to successfully double a more complex structure in all three dimensions.

5 I understand and can easily experiment with many different types of sophisticated quantitative relationships, and am able to readily and fluently explore these relationships using mathematical concepts like ratios and proportions. I can apply ratios and proportions in ways that allow me to fluidly accomplish whatever goals I set for myself at an exhibit.

For example, at *Blocks* visitors with this fluency aren't just thinking of doubling in three directions; they also realize that they are doubling area (in two directions) and volume (in three directions. They can answer questions like "How many blocks do I need to double this structure in three dimensions using mental math?".

Most respondent groups had at least some members (especially, but not always the younger members) struggle with even simple ratios like doubling (especially if they had not yet studied multiplication at school). Even many adults had trouble recognizing ratios and proportions at exhibits like *Clicker* and *Spirograph*.

For most groups at most exhibits, recognizing and exploring even some of the more basic quantitative relationships proved difficult and particularly challenging. However, at some exhibits, many recruited respondents explored at least some quantitative relationships. This was especially true at the doubling exhibits, for example *Blocks*, *Chairs*, and with some of the challenges at *Shadow Table* and *Big Shadows*. (In comparison, only a few casual respondents were observed [for example] doubling structures at *Blocks*, or drawing 2:1 lines at *Sliders*.) It was also interesting to note that RRs were more apt to double in three dimensions at *Blocks* than at *Chairs*, presumably because at *Blocks* this was a challenge you could explore with concrete materials, but at *Chairs* you had to multiply some pretty big numbers in your head.

In some cases, the quantitative relationships were complex and rarely discovered, such as at *Clicker*, *Rainbow*, and *Spirograph*. The quantitative ratios/proportions at the core of these exhibits were difficult for even adult visitors to find and use. For instance, most respondents were not thinking in terms of gears at *Spirograph*, and none discovered the relationships between gear ratios and patterns (which are pretty complex even once you do recognize the gear ratio idea). One RR adult seemed to be getting close to this kind of understanding when he made some early-on connections between *Spirograph*, *Shapes from Circles*, and the hula-hoops featured on the photomontage. But he never included gears or gear ratios in his discussions, and he never followed up on this during any of the later visits (C2-1). Recognizing the application of proportions at *Clicker* also required pretty sophisticated thinking. And at *Rainbow* although most RRs seemed to have made a link between the colors on the rainbow floor mats and the color bands on the screen, and many developed a high level of kinesthetic fluency at the exhibit, many did not fully grasp the significance of the horizontal (or X) axis as a representation of time, or a history of where they had been.

Overall, many respondents—when they engaged with the exhibits quantitatively—increased their fluency with understanding quantitative relationships such as doubling. Respondents who engaged with exhibits exclusively in qualitative or kinesthetic ways tended not to increase their quantitative fluency (for example, see the section below *Visitor Vignette: Jessie & Estelle*).

3.4e Using Numbers and Other Quantitative Tools at the Exhibit

All of the *Math Moves!* exhibits incorporated numbers[12] in various ways. The focus of this hierarchy is on how respondents recognized and used these numbers that were incorporated into the exhibits as part of their engagements. It should be noted that the use of numbers was not a requirement for engaging successfully at most of the exhibits. Numbers were however incorporated into the exhibits to provide visitors additional opportunities for engagement and understanding.

0 I'm not thinking about numbers at this exhibit. I don't know what the numbers are for, and I'm not interested in them. I may not even see the numbers.

1 I wonder what the numbers are for, or I wonder how to use these numbers to enhance my experience with this exhibit.

2 I think I know what these numbers are for and how to use them, or I partially know. My understanding may be limited, naïve, or incomplete. Or my understanding may be inaccurate. For example, I may think that how high I jump at *Rainbow* makes the graph line steeper. Or I may think that the numbers on the paddles at *Clicker* represent fractions. I may notice the numbers on the *Spirograph* wheels, but I'm using them purely as identification labels, without recognizing their values as numerical constructs.

3 I understand the basic use of the numbers at the exhibit. I am able to successfully use (at least some of) the numbers to enhance my experience at the exhibit, and to develop my understanding of ratio, proportion, and related math concepts.

[12] In this case, the term "numbers" refers to all the related numerical aspects including graphs, slopes, rates, octaves, etc., in addition to the actual numbers.

Respondents at Level 3 successfully (for example) used a measuring device at *Chairs*, used the grid numbers at *Shadow Table*, and counted the number of blocks they were using to create their sculptures at *Blocks*.

4 I'm pretty fluent with using numbers to enhance my exhibit experience. I use the numbers in a variety of ways to explore, and enhance my understanding of ratio, proportion and related math concepts.

5 I readily and fully use the numbers in the exhibit in many different (and increasingly sophisticated) ways to enhance my understanding of ratio, proportion, and other mathematical relationships.

During the initial study visits, most respondents spent little time using the numbers at most exhibits. One exception to this was at *Blocks*, where there was counting going on, but it was often not about doubling. Another exception was at *Chairs* where some RRs did quite a bit of measuring (and therefore using numbers) during their early visits.

In general, respondents ranged greatly in their use of numbers and numerical tools, especially at different exhibits. Most respondents to most exhibits did not notice the numbers until their second or third, or even subsequent use of the exhibits. And once they noticed the numbers, many respondents could not figure out what they were supposed to do with them. For example, the retractable string at *Shadow Table* proved particularly baffling to some respondents, if they noticed it at all. Many respondents ignored the numbers on the paddles at *Clicker*, and when they did notice them, they were confused by how to use them.

At some exhibits like *Spirograph*, most respondents didn't notice or use the numbers at all (Level 0). When they did use the numbers, they used them as labels for the disk that they wanted (Level 2). On the other hand, at exhibits such as *Balance*, most respondents quickly picked up and played with different quantitative relationships by using the numbers on the cubes and doing simple addition or multiplication (Level 3). At *Chairs* many respondents successfully measured them using either the measuring tape or the foam core squares (or both). Most respondent engagements at *Shadow Table* did not include the use of the numbers, grid lines, or retractable string to explore ratio and proportion, and at *Sliders*, most respondents engaged with the numbers in basic ways, such as making stair steps, but not exploring more sophisticated ratios. Most groups had at least some younger members struggle with even simple ratios like doubling (especially when they had not yet studied multiplication at school), and even many adults had trouble recognizing ratios and proportions at some exhibits, especially *Clicker* and *Spirograph*.

Once respondents started measuring at *Chairs*, there were plenty of interesting discoveries to think about. For example, some respondents first discovered that the medium chair is twice as tall as the small chair, and the big chair is twice as tall as the medium chair, but four times the height of the small chair (Level 3). They then figured out that the area of the medium chair's seat is four times the area of the small chair's seat, and the big chair's seat is 16 times the area of the small chair seat (Level 4). And when considering volume, a few went on to discover that it grows at an even faster rate from the small to the big chair: from 1, to 8, to 64.

It was interesting to note that when they didn't notice or use the numbers at an exhibit, many RRs indicated they felt they were done with the exhibit because they saw nothing new to do and became bored, in spite of the fact that they had not yet discovered some of the additional (numerical) ways of engaging. At the end of the study, when the researcher took the liberty to point out some of these additional opportunities involving ratio and proportion, the RRs quickly became re-invested in the exhibits and excited about spending more time exploring them in new ways. "[After you pointed them out] all of a sudden I was seeing all the numbers for the first time today and realizing that there was some relationship between the numbers. I never noticed them before." (C2-6) A few RRs specifically requested that "hints" be included in the exhibits, to draw attention to some of these numerical opportunities.

Overall, the data indicated that most recruited respondents did increase their fluency with using numbers at at least a few exhibits, especially at *Chairs*, *Blocks*, and sometimes at *Shadow Table* when RRs counted the squares on the shadow wall to compare the size of two shadows. At most of the other exhibits, most respondents tended not to significantly increase their fluency with the numbers over time. Most settled on a particular way of using numbers (or not using them) at a particular exhibit, and pretty much stuck with that way over the expanse of their study visits. However, when the numbers were pointed out to them during their final study visits, they quickly increased their fluency.

Few casual respondents were observed using numbers or quantitative tools at most exhibits. This is not surprising, considering that RR groups often took at least a visit or two to start noticing or using the numbers.

All of the hierarchies described above describe the ways visitors could evolve their fluency over time. Two additional hierarchies deserve mention as well. These hierarchies represent (f) different ways visitors understood the main point of the exhibit, and (g) visitors' appreciation of math, ratio, and proportion as part of everyday life.

3.4f The Main Point of the Exhibit

Although we researchers told the recruited respondents up front that they would be using exhibits about math, we did not tell them that the exhibits were specifically about ratios and proportions. We left them to figure that out on their own through their engagements with the exhibits and the signage developed by the exhibit designers. A few (adult) respondents figured that out on their first visit, first for one exhibit, then another, eventually realizing that all the exhibits included ratios and proportions in one way or another. In other cases it took a lot longer, as some adults gradually noticed the common mathematical theme, or as older children's mathematical understandings developed to the point that they understood more about the concepts of ratios and proportions. This hierarchy describes the range of ways respondents understood what the exhibit was about.

0 I don't know what this exhibit is about, and I'm not really interested. I may not even really be aware of the exhibit per se; it's merely backdrop to something else I am doing.

1 I wonder what this exhibit is about.

2 I think I know what this exhibit is about, or I'm just starting to figure it out. My understanding of what the exhibit is about may be inaccurate, or if not inaccurate, unsophisticated or only partially true. For example, I may think *Spirograph* is primarily about drawing interesting pictures. Or *Rainbow* is about racing with my friend.

3 I get the basic gist of what the exhibit is about. I'm probably not fully aware of all of it, but I know it's about math, making comparisons, and finding relationships; that is, I know it's about ratio and proportion – although I might not use those words.

4 I understand that the exhibit is about exploring the world of ratio and proportion in a variety of ways – again regardless of whether or not I use those terms.

5 I fully understand that the exhibit is about many different ways of exploring ratio and proportion – even though I may not use those terms. I understand that it's a platform for repeated exploration of these concepts in sophisticated ways.

Even though recruited respondents were told that these were exhibits about math, many (especially children) seemed to forget that at various times during the study. Some respondents evolved their understanding of the main point of the exhibits, or slowly came to realize that the exhibits all shared the common theme of ratio and proportion. Some of the younger children never reached Level 3 for most exhibits or for the exhibition as a whole, although they usually had a basic (Level 3) understanding for those exhibits that were specifically about doubling. Some adults also never reached Level 3 for the whole exhibition, although they usually recognized that at least a few of the individual exhibits were about math, making comparisons, and finding relationships.

Whether it's important that visitors think about and talk about the exhibits as math, or whether it doesn't matter as long as they are engaged in math activities, is an issue that was discussed among the project staff (see discussion on Content vs. Experience in the section below: What Was the Experience of Professionals?) with no final resolution. It did seem to play a role in increasing respondent fluency in quantitative relationships, and in using the numbers; when respondents didn't see the math, they tended to increase their quantitative fluency to a lesser degree.

3.4g Appreciation for Math, Ratio, and Proportion as Part of Everyday Life

This hierarchy describes visitors' attitudes towards and enjoyment of math, as well as their recognition and use of it as part of their everyday lives. While helping visitors develop positive attitudes towards math was not a direct goal of the *Math Moves!* exhibits, the project team was interested in seeing if the exhibits helped visitors expand their perceptions of and attitudes towards math.

0 I don't know about or care about math.

1 I'm curious about math.

2 I know what math is, but I don't like it; it's boring, or it's too hard. Or I think I know what math is all about, and I may like it, but my understanding is limited or uni-dimensional. For example, math is what I do second period at school.

3 Math is fun, interesting, and part of my everyday life, whether in or out of school.

4 I like to recognize and use math, ratio, and proportion to solve basic everyday challenges such as dividing up a pizza, or figuring out how fast something is going.

5 Math enables me to explore and wrestle with all kinds of interesting and complex problems in everyday life. I especially love when I can challenge myself mathematically, or I am able to use math to solve a complex everyday problem or to think differently about something in my daily life.

Respondents for this study were purposively selected to include a wide range of attitudes towards math; a few already had very positive attitudes towards and appreciation for math in their lives (Level5), there were also some who had a less positive, or more limited attitude (Levels 1 and 2), as well as some tag-along younger siblings at Levels 1 or 0. With a few notable exceptions, the data indicated that most RR's appreciation of math did not tend to change very much during the course of the study. A large part of the reason for this may be attributed to the fact that (as described above) so many of the RRs tended to enjoy and use the exhibits without thinking that much about the math.

While a few respondents who had a less positive attitude towards math increased their positive perceptions about and feelings toward math over the course of the study (for example, see the description and quote above in the section *Basic Math Skills*), for most respondents, the data indicated little shift in attitudes. It may be that this effect will only show up over much longer periods of time as children take more math in school and are able to think back and reflect on their experiences in the exhibition. And when they did see the math, many RRs struggled with being able to connect that math with much in their daily lives.

On the other hand, we found no evidence that engaging with the *Math Moves!* exhibits contributed to any respondents leaving the study with *less* enthusiasm or passion for math than when they arrived. A few respondents did become bored with many of the exhibits when they felt they had run out of things to do with them but this didn't appear to influence their feelings about math. This is in contrast to the findings in the summative evaluation findings for the *Handling Calculus* exhibit that found that the exhibit brought out negative or anxious feelings. This is most likely because the *Handling Calculus* exhibit was more overtly about math, and reminded people clearly of difficult classroom experiences where they struggled with learning calculus concepts (Gyllenhaal, 2006).

Summary & Discussion

As stated earlier, this study was particularly interested in how respondents' fluency with math, ratio, and proportion evolved over time. Each of the seven hierarchies described above show different aspects of this. There was great variability among visitors, at different exhibits, and even at different museums. For the most part however, respondents appeared to gain the greatest fluency over time with (a) qualitative and kinesthetic understandings of ratio and proportion, and (b) the operation and use of the exhibits. There was some increase in fluency with both (c) using basic math skills, and (d) understanding quantitative relationships. There was the least amount of change over time in respondents' (e) successfully using the numbers, (f) recognizing the exhibits as being about ratio and proportion, and (g) appreciation of math, ratio, and proportion.

Qualitative and Kinesthetic Understandings of Ratio & Proportion

Making simple qualitative comparisons seemed to be something that all respondents did quickly and easily (Level 2). It was fun to watch respondents move their bodies in interesting and creative ways as they explored the exhibit, experimenting with how they could manipulate shadows, graph lines, sounds, etc. Most respondents increased their fluency from an initial Level 2 to a Level 3, and some increased to Level 4 fluency at some exhibits, especially at *Big Shadows*, *Rainbow*, and to some extent *Sliders*. Some respondents were limited in increasing their fluency by the mechanical operation of the exhibit such as *Clicker* not staying in sync, or when they got side tracked by the lines dropping out at *Rainbow*. And a few respondents increased their kinesthetic fluency to Level 5 at a few of the exhibits.

Engaging with and Using the Exhibit

When it came to looking at how respondents engaged with and use the exhibits, all respondents began at Level 1, wondering what the exhibit was about and what they could do at the exhibit. By the end of the final study visit, all respondents had increased their fluency with the mechanical operation of most of the exhibits, such that they were able to quickly and easily use most exhibits at a basic Level 3, i.e. in a way that would help evolve their understandings of ratio and proportion. Of the nine core exhibits, this was especially true at *Rainbow*, *Sliders*, *Shadow Table*, *Blocks*, *Balance*, and *Chairs*, and less so at *Spirograph*, and *Clicker* where most respondents tended to use the exhibits at primarily Level 2. Respondents rarely returned to *Comment Board* a second time so it was not possible to assess an increase in fluency there. Some respondents used some exhibits in more sophisticated (Levels 4 and 5) ways including searching out, trying, and coming up with their own new challenges. On the other hand, at some exhibits some respondents quickly evolved their fluency with ratio and proportion, but then assumed they were done, and either avoided returning to the exhibit, or if they did return, reverted to using it in a way unrelated to math, ratio, and proportion.

Using Basic Math Skills

All RRs arrived at the exhibits with some basic math skills, but these skills varied greatly. With children this often depended on the age of the respondents and where they were in school, but there was also great variability in adults' comfort and experience as well. At the beginning of the study, most RRs had the basic skills necessary to use most of the exhibits (i.e. Level 3: counting, adding, subtracting, and measuring), but some of them (including some adults) struggled (Level 2). Over the course of the study, as many children's fluency in these basic skills increased, and as they learned new skills at school (e.g. multiplication, division, fractions, graphs), they were able to use the exhibits to explore ratio and proportion more easily. While most child respondents' basic math skills fluency increased over the course of the study, it is difficult to tell the role the *Math Moves!* exhibits played. Some respondents clearly became more fluent using the various measuring devices at *Chairs* in particular, developing more fluid movements and making fewer errors. Over the course of the project, most respondents began at Levels 2 and 3, and increased their fluency by about one level.

Understanding Quantitative Relationships

Most respondents arrived at the exhibits at Level 0 at most exhibits, with minimal thinking about ratio and proportion, or making comparisons. They quickly gained fluency in recognizing quantitative relationships, especially at exhibits like *Chairs* where it was virtually impossible to not notice the difference in size. Most quickly reached Level 3 fluency where they noticed and were able to calculate basic quantitative relationships at many of the exhibits. At *Rainbow*, understanding quantitative relationships was more difficult, as the designers of this exhibit in particular tended to focus primarily on qualitative and kinesthetic understandings. Few respondents indicated a Level 3 fluency in quantitative relationships at *Rainbow*, but most were at Level 3 by the end of the study at the other exhibits.

Using Numbers and Other Quantitative Tools at the Exhibit

Most respondents began their initial study visits at Level 0, i.e. not being aware of the numbers at most of the exhibits. At *Chairs* in particular, most quickly discovered the measuring devices and were able to incorporate them into their engagements, increasing their fluency to Level 2 or 3, and even in a few cases Level 4. At some exhibits, even at their final study visit, most respondents showed little evidence of fluency in using numbers. Most respondents did not gain fluency in using the numbers beyond Level 2 at most of the exhibits, except *Chairs* where counting and measuring were readily accessible to most visitors, and at *Balance* when numbers were used to balance the scale.

The Main Point of the Exhibit

Even though the respondents were told at the beginning of the study that the exhibits were about math, most did not appear to think of them in that way during their engagements with them. A few respondents (especially adults) figured out that they all shared the common theme of ratio and proportion. Most respondents however started and ended the study at Level 2, thinking of the exhibits as fun activities to do, all loosely clustered around the theme of math.

Appreciation for Math, Ratio, and Proportion as Part of Everyday Life

Because so many respondents perceived the exhibits to be only peripherally related to math, it was often difficult to assess their feelings about math. Most respondents began the study at Level 2, i.e. a limited view of math, math is something that is done primarily at school, or math has little to do with everyday life; and most were at the same place when the study was completed. There were few indications that participating in this study or engaging with the exhibits helped them think more about math as part of their everyday life except for a few instances when an exhibit reminded someone of using a hula-hoop, or balancing on a teeter-totter. Some of the ways that engaging with these exhibits will contribute to respondents seeing math, ratio, and proportion as part of their everyday lives will likely take place only much later on, at some time when they reflect back on or are reminded of something they did in the exhibition (Perry, 2002).

Discussion

So, what contributed to these kinds of evolutionary experiences and increases in fluency when they happened? In this section we will briefly examine three aspects: the embedded exhibit design challenges; the exhibit signage and labels; and facilitation by adult caregivers.

One thing that contributed to increased fluency was when the exhibit was designed in such a way that there were increasingly sophisticated challenges. A good example of this was at both *Chairs* and *Blocks*. These two exhibits worked well at helping visitors evolve their understandings and engagements because visitors could do and think about them in one, two, or three dimensions. Thus even 6 and 7 year olds could successfully see "double" in one dimension at *Chairs*, and they could double in three dimensions at *Blocks*, even if they only thought in terms of doubling one direction at a time (wider, then longer, then taller). On the other hand, once they were thinking in three dimensions, there were lots of cool surface area and volume effects (like running out of blocks when they tried to double the doubled structure).

During the initial study visits, many of the exhibit challenges on the labels also contributed to RRs' fluency. In these situations, many adults read the challenges and used them to help both themselves and their children discover qualitative and quantitative relationships and, less often, engage with the numbers. A few RR children read the challenges on their own, especially during later study visits, and used them to engage with the exhibits. The illustrated challenge labels at *Rainbow* effectively engaged visitors in increasing levels of difficulty as they progressed from the first challenge to the second and so forth.

However in many instances, the challenge labels proved overwhelming for many RRs. For example, at the *Shadow Table*, many RRs started their engagements with the first challenge "How can the smallest wolf be twice as tall as the biggest tree?" Many RRs tried to immediately start with this challenge without first of all playing with and getting a kinesthetic feel for what happens when you move the objects closer to and further away from the light. They tended to get stuck trying to do that challenge because it was the first thing they saw, and then they quickly became overwhelmed and frustrated, ultimately abandoning the challenges altogether.

If on the other hand the challenge had started with a simpler task, even something such as "Make the bunny bigger." or "How tall is your bunny?" i.e. a task to jumpstart visitor engagement in the right direction. Even a simple challenge like this however, needs to be tested on visitors to see if it does in fact get them playing with the figures and shadows and paying attention to size, the first step in successfully engaging with ratio, proportion, and embodied cognition.

At *Clicker*, there were similar frustrations with the initial challenge of "How would you get wheels of different sizes to click at the same time?" This was a very difficult challenge for many visitors, even when the exhibit worked flawlessly—which it rarely did.

While the label challenges in many cases helped some of the RRs evolve their understandings of ratio and proportion, for most of the focus children, a primary contributor to their ability to increase their fluency was support and guidance from adults. This support included directing attention to important parts of the exhibit, suggesting challenges they might want to pursue, providing guidance and explanations when needed. For example, many RR adults encouraged children to notice the shadows at *Shadow Table*, or watch the screen at *Rainbow* as they moved back and forth. These support and guidance behaviors helped children start to think about relationships and making comparisons, and in some cases to notice the numbers embedded in the exhibits.

During later study visits most recruited parents said they were less involved with their children at the exhibits, and that may be one explanation for why fluency with quantitative relationships and with using the numbers tended to not increase over time. Some research has indicated that the reason for lack of parental involvement in situations such as this is not due to a lack of a desire to interact with their children, but rather a lack of knowledge about how to do so, especially when parents are unsure about the exhibit themselves (Perry, 2012, p. 21-25).

During the final study visits, the researchers sometimes gave quick hints and brief explanations at many of the exhibits (such as pointing out the numbers on the *Clicker* wheels), trying to see if RR adults and children could successfully use ratios and proportions to meet exhibit challenges *if* they had additional (but basic) support. These instances of directing attention to important (but often overlooked) parts of the exhibits and providing brief explanations, helped some RR families at some of the exhibits evolve their understandings and fluency even further, and often helped them move beyond boredom and frustration.

The data indicated that while helpful in some cases, challenge questions alone weren't enough for many visitors. Some RRs need additional hints or more explanations (or both) about the math. Another area where many RRs got hung up was when they needed to know how the exhibit worked before they could begin to engage in any mathematically meaningful way, for example, knowing where the electric eye on *Rainbow* was, and what it was reading; recognizing the gears and their role at *Spirograph*; and, at the shadow exhibits, for example *Shadow Table* and *Big Shadows* learning some basic concepts related to light and shadow (such as rays of light travel in straight lines).

It should be noted that the exhibit labels were designed specifically to avoid giving detailed explanations or instructions because so often these type of labels tend to shut down visitor experiences rather than open visitors to pursue further exploration. McManus (1990, p.5) talks of labels as too often being conversation "hogs," monopolizing the conversation and something to be avoided just as you would avoid someone at a cocktail party who talks too much.

We are not recommending the inclusion of detailed explanatory labels about the science and engineering behind the exhibits, but rather strategic (and fully tested) read-at-a-glance hints to direct visitors' attention to important parts of the exhibit they may be overlooking. These hints can serve to get visitors headed in the right direction. Examples might include an identification label on the sensor at *Rainbow*, a suggestion to "Count the teeth" on the *Spirograph* gears, and so forth, i.e. tools to *start* visitor conversations not shut them down (Perry, 2012, p. 92).

A slightly different strategy was used at the math exhibition *Design Zone* where they developed what they called *parent labels* specifically aimed at helping parents find the math in the exhibit so that they would be able to help support their children as they tried to meet the exhibit challenges (Garibay Group, 2013). Finding the right balance of providing the appropriate hints and guidance without over-burdening visitors is difficult (but not impossible) to accomplish. It requires a large amount of rapid prototyping until the right balance is hit upon.

This section set out to describe in some detail the various ways *Math Moves!* respondents increased (and in some cases didn't increase) their fluency along a number of different dimensions. While most respondents increased their fluency in using the exhibits, and in engaging with ratio and proportion in qualitative and kinesthetic ways, there was less evidence of increased fluency in quantitative understandings and engaging with ratio and proportion in quantitative ways. Child respondents did appear to increase their fluency with basic math skills, probably to a large extent because of learning math in school, but most adults did not appear to.

3.5 Visitor Vignettes

One of the more interesting opportunities from this study was being able to follow each of the respondent groups across all of their study visits. In this section we include a detailed look at two such groups, and only regret that we are not able to include more. (Note that all names have been changed.)

Vignette #1: Molly & Chris - Using Numbers to Explore Ratio and Proportion

The following vignette is an interesting look into one respondent group's engagements with the *Math Moves!* exhibits over the course of the study. This respondent group was selected because they worked very hard to engage with the many numerical and quantitative aspects of the exhibits. Their close working relationship as they collaborated to solve various challenges was also notable although at times they said they struggled with how to talk about math.

The focus child, Molly, was 7 ½ years old when she was recruited in early spring 2012, and she was in 2^{nd} grade at a public school at that point. By the end of the study she was 9 years old and had been in 4^{th} grade for a bit more than a month. Her mother, Chris (in her 30s) brought Molly to *Math Moves!* each time. Usually Molly and her mom came alone, but on one study visit Molly's brother (early teens) came along, as well as Molly's friend from school. (See Table 3.1 for a summary of their study visits.)

During the recruitment interview Chris said her daughter liked math. "She's in the high math group at school. But she is sometimes hard on herself and that creates difficulties for herself….She likes word problems. She enjoys math but sometimes gets a little frustrated. But when she gets the answer, it's fun."

Talking about herself during the recruitment interview, Chris said, "I always enjoyed math but don't use it a lot other than normal day-to-day kinds of things. It sometimes took me a while to get it, but I did get it. Math was one of the 'funner' subjects for me. [I] took math only through Algebra." They had received a museum membership as a gift about one year before the start of the study and came to the museum three times the previous year, but that membership was about to expire. Chris said that with the membership they received as respondents "It will be fun to explore more of the museum, and good for [Molly] to focus on math."

During the initial study visit Molly seemed bright and lively and open with her feelings. During the interim visits, Molly and her mom never got to all of the *Math Moves!* exhibits during a single visit. Chris was very conscientious about taking photos during the interim visits and then e-mailing them to the researcher shortly afterwards. She did a good job of documenting her daughter's engagements at the exhibits.

During the initial study visit, both Molly and Chris quickly settled in on a pattern of engagement. Once they were at an exhibit, Molly often messed around for a bit—for instance, she made hand shadows at *Shadow Table* and pretended to knit with the mobile rods at *Balance*. While Molly was exploring in her own way, her mom read the labels, looked over the different parts of the exhibit, and watched what Molly was doing. Once Chris felt she knew what they were "supposed to do" at the exhibit, she stepped in and took a more active role in the engagement. She usually took on something of a teacher role by directing or focusing Molly's attention, asking questions, posing challenges or things for Molly to do, and giving feedback and encouragement. Chris pretty much maintained this role until Molly was using the exhibit as the labels suggested it was intended to be used.

As the first visit continued, Chris worked hard to help Molly understand what was going on at each exhibit, sometimes simplifying the interpretation and challenges so that Molly would (for example) notice what happened to a shadow at *Shadow* Table when a piece was moved closer to the light, before encouraging her to try the exhibit challenges (like making the shadow of one piece twice as high as another piece's shadow). There was lots of conversation between mom and daughter at all of the exhibits, and much of it centered on Chris teaching and Molly being the student. However, there were also times when they worked together to meet the exhibit challenges or make up challenges of their own. The two also engaged emotionally in many ways, laughing together and generally having a good time. Molly often worked to get her mom to laugh and sometimes sought approval for the products she produced.

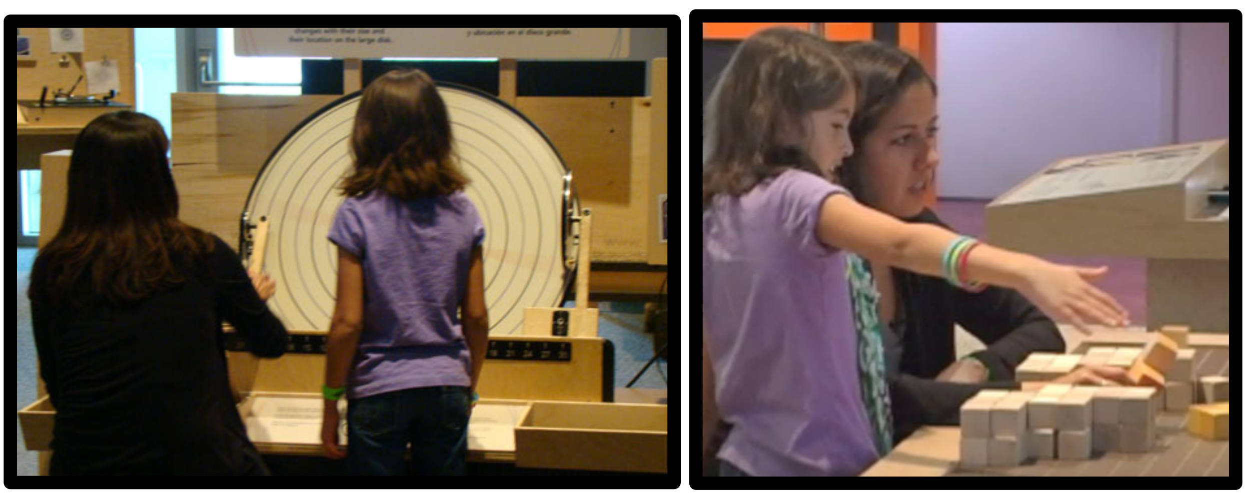


Figure 3.10: Chris generally took on a teacher-like role as they figured out each exhibit. However, Chris and Molly worked together to meet some challenges during study visit one.

The interactions between Molly and Chris varied across the different study visits (Table 3.1). During some interim visits Chris was in teacher mode, but on others Molly was more on her own. For instance, during the second visit Chris said she "let Molly play more," than she had during the initial visit, exploring and having fun is ways that were not described in the exhibit labels. For the third visit, Chris said that, based on Molly's asking where the math was in the exhibits, they made it their mission to find the math in this exhibition. So, Chris was in teacher mode again during this visit—studying the labels, posing the label challenges for her daughter, and helping her work through them using math tools at the exhibits—but she said that Molly still decided which exhibits to go to and how long to stay. During the fourth visit mom and daughter continued to work together pretty closely, and Chris kept her daughter focused on the exhibit challenges—completing them rather than "just having fun" by exploring on her own. Chris admitted that Molly got pretty frustrated with her mom directing her to focus on the challenges rather than exploring freely, and she was ready to leave *Math Moves!* after stopping at only four of the exhibits. During the fifth visit Molly worked more independently because there were two other children in their visiting group, and Chris felt she needed to keep an eye on all of them. During the final study visit, the researcher asked Molly to give a tour of the exhibits, showing what she could do at each one. Molly initially took the lead at most exhibits and demonstrated what she understood about each exhibit. Then Chris took her turn. She did some teaching of Molly at each exhibit, and she also asked Molly what she liked and didn't like about the exhibit they had just used.

Study Visit	*Length of visit in mins.*	*Who visited*	*Description of Visit*
#1	90	Molly & Chris	Initial visit. Molly worked closely with her mom. Researcher was on site.
#2	40	Molly & Chris	Molly used the exhibits primarily by herself.
#3	70	Molly & Chris	Looking for the math. Molly worked closely with her mom.
#4	30	Molly & Chris	More looking for the math. Molly continued to work closely with her mom.
#5	25	Molly, Chris, Molly's brother, a friend	Molly used the exhibits primarily by herself as her mother kept an eye on all three children.
#6	80	Molly & Chris	Final visit. Molly gave the researcher a tour of the exhibits, explaining what she could do at each of them.

Table 3.1: The different study visits varied quite a bit in terms of length of visit, as well as the interaction between Molly and Chris.

From the initial visit, mom and daughter included the challenges described in the exhibit labels as a major part of their experience. Chris was the primary instigator for this, once she read the labels and started working with Molly at each exhibit. During the third and fourth study visits, when they focused on *finding the math*, Chris said they tried to answer the challenge questions at most of the exhibits. Even on the visits where Molly worked mostly on her own however, (i.e. the second and fifth study visits) Chris reported that Molly remembered many of the challenges from previous visits, and attempted them on her own, sometimes varying them in small ways (e.g. lining up the colored squares at *Shadow Table* on the 2:4:6 lines instead of the 2:4:8 lines she had put them on with her mom's help during an earlier visit, photo below). During the final study visit they did one or more of the exhibit-label challenges at almost all the exhibits, with Molly taking the lead.

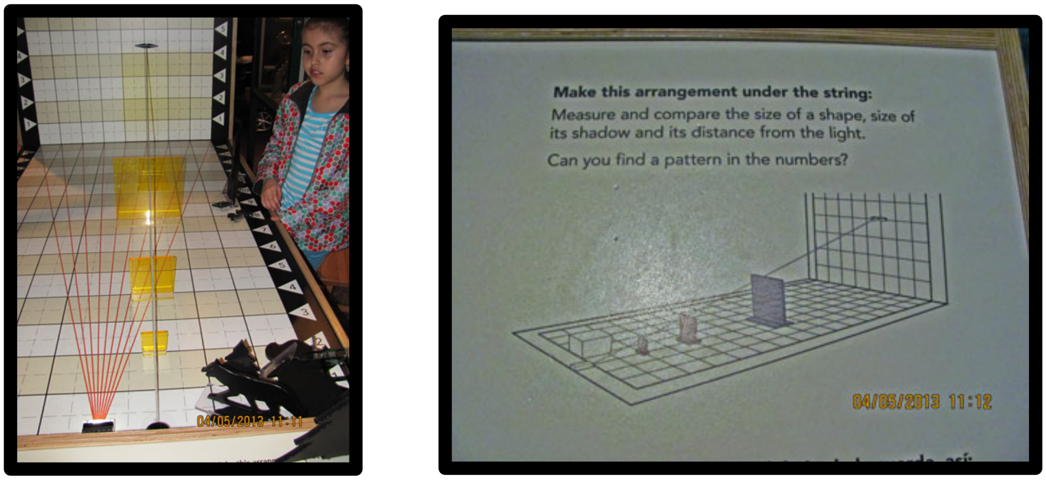


Figure 3.11: During one visit, Molly completed an exhibit challenge using the string and plastic squares at *Shadow Table*.

This respondent group, more than most, focused strongly on the numerical math at the exhibits. This was likely due (at least in part) to their focus on the exhibit challenges. Chris and Molly used progressively more numerical operations over the course of the study. Molly used more mathematical operations during the visits when she worked closely with her mom, but she still tried to use some during the visits when she worked more on her own.

During the third visit Molly began using some of the multiplication skills she had been learning at school, and during visit four Chris took every opportunity to get Molly practicing her multiplication skills and multiplication tables.

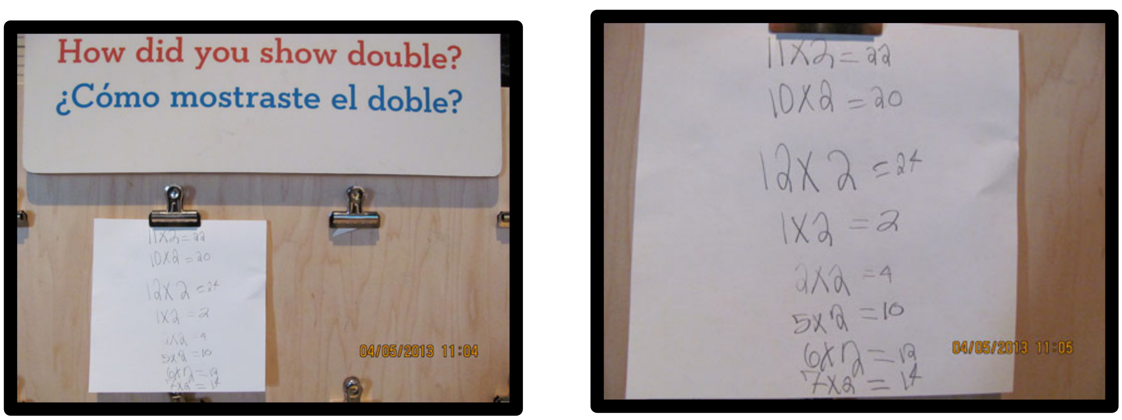


Figure 3.12: During Visit 4 Molly practiced her multiplication tables (something she was just beginning to learn at school) at several exhibits, including the *Comment Board*.

Molly and Chris increased their numerical fluency in ratios and related concepts than most other respondent groups. For instance, Chris invented a way of doubling the colored models at *Blocks* by first making cubes using eight blocks (2 x 2 x 2), and then fitting the cubes together to make the doubled structure. She taught this to Molly, and during a subsequent visit, Molly taught it to her friend and brother.

This group was also one of only two RR groups to discover on their own how to quantitatively apply ratios at *Clicker*. In addition, Chris figured out a way to explain the ratios to her daughter using only multiplication, since Molly had not yet learned division. For example, instead of saying that the 6 wheel on the 24, and the 3 wheel on the 12, both reduced to the same ratio of 1:4, she just pointed out that 6 times 4 is 24, and 3 times 4 is 12. (That way she did not have to introduce the word *ratio*, which Molly did not know). It's interesting that this happened even though Chris had said she did not feel she was particularly accomplished with math.

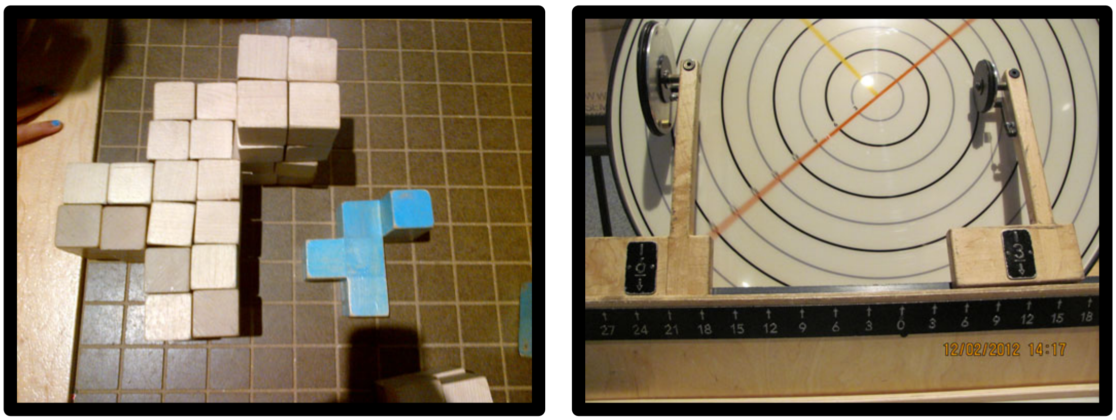


Figure 3.13: Molly duplicated a complex block model using Chris's technique (Visit 5), and Chris and Molly made the wheels click at the same rate using ratios and proportions (Visit 3).

During the study, Molly and Chris increased their fluency along numerous dimensions, although it tended to happen in a stepwise rather than continuous fashion. For example, the initial study visit was mostly about figuring out the mechanics and how to do things at the exhibits. These first steps were important, but there was little time for repetition and practice (which appear to be an important component for developing any type of fluency). During the second visit, Molly seemed to become more comfortable with the mechanics of the exhibits, and with the basic physical things they were "supposed to do." Molly became more fluent at developing a kinesthetic and qualitative understanding of ratio and proportion at some of the exhibits during visit two. Probably because of the third visit's focus on finding the math, Chris and Molly both slowly increased their fluency with basic math skills, and became more comfortable using them to complete the exhibit challenges, including using numbers and exploring quantitative ratio/proportion operations at several of the exhibits. During the fifth visit there appeared to be a fluency plateau of sorts, as Molly's mom did not work closely with her.

By the final study visit, both mother and daughter were pretty fluent with the mechanics of most exhibits, although Molly seemed a bit rusty using some of them (e.g. it had been almost 10 months since she had last had a chance to use *Spirograph*, and so she still had difficulty assembling the mechanism and changing the gears). Molly was also becoming quite fluent with some aspects of the math skills, kinesthetic understandings, using numbers, and understanding quantitative relationships, like doubling at *Blocks* and the *Comment Board*. However, she had not yet developed fluency at applying multiplication with large numbers, as she needed to do at several exhibits (like *Chairs*), and she had forgotten how to use the ratios to solve the *Clicker* challenges (which she had not worked on together with her mom for 10 months; her mom did not remember either.)

Throughout the study, both Molly and Chris expressed positive feelings about most of the exhibits and about the exhibition as a whole. Molly said her favorite exhibits were *Spirograph*, *Rainbow* and *Sliders*; these favorites developed early and were maintained through the end of the study. Chris said that, with Molly's favorite exhibits, each time they came they would learn a little more about it. "I think the more time you spend with it, the more you'll understand, not necessarily how it's related to math, but how the exhibit is supposed to be used."

However, Molly and Chris also got frustrated with a couple of exhibits that didn't work well at times (for example the difficult-to-turn cranks at *Shapes from Circles*), and with exhibits where it was hard to complete the challenge (like drawing the circle at *Sliders*). Chris also expressed some frustration with the interpretive labels at the exhibits. She talked about how she had trouble explaining the exhibits to Molly and finding the right words to talk with her about the math. For instance, Chris said that at *Shadow Table*, moving the cutout closer to the light and having its shadow get bigger was a pattern (as pointed out in the label)—and that pattern is math.

However, she said that *finding* the patterns was one thing; *explaining* the patterns was something else—and she did not know how to explain the patterns that they discovered. Chris said, "I'm not illiterate in math, but a lot of things I wouldn't know how to explain to her what was so mathematical about it all." She said she wished there was something that "explain[ed] more what it has to do with math and maybe relating it to things that you do in the world… just a little bit more instruction or guidance on the actual way it deals with math."

Asked if their feelings about math changed because of the exhibits, Molly said "Not really. I kind of always liked math." Chris said, "I don't think it changed my feelings at all towards math… it didn't make me like math any more; it didn't make me like math any less. I mean, when I was in school I liked math. I wasn't that great at it, but it was always kind of more of a fun subject." Asked if they liked the exhibits more before or after they "found the math" during visit three, Molly said, "I think I like it more, that we know what to do, and we know more about it and know more math about it. And so it seems funner to know that stuff." Chris said, "I think it's funner with the math because, like I said, I try to relate it to what she's learning in school and try to pick up on it and ask her questions that she would be able to solve within the station."

Vignette #1: Jessie & Estelle - Coming Up With Their Own Challenges

This next vignette follows a family at a different museum as they engaged with the *Math Moves!* exhibits. This group took their role as respondents in the study very seriously, and worked hard to get as much out of the exhibits as possible. In spite of their diligent hard work, unlike the respondents in the previous vignette, these respondents increased their fluency with ratio and proportion only minimally. They did however engage in other aspects of math such as counting and spatial reasoning, and they enjoyed their time in the exhibition. They were the only respondent group to repeatedly return to the exhibition not as part a study visit.

The focus child, Jessie, was 7 years old when she was recruited in early spring 2012, and she was finishing up 1st grade at a public school at that point. By the final study visit she was 8 ½ years old and was about to enter 3rd grade. Her mother, Estelle (in her 30s) brought Jessie to *Math Moves!* each time. Jessie's younger sister also came along on several visits, and her older sister, father and aunt also got chances to experience *Math Moves!* during study visits.

Jessie and her mom both spoke English most of the time in the museum. Jessie was in the bilingual class in 1st grade, where she learned math in Spanish. Jessie said it was easier for her to count in English; however, there were times when Estelle translated researchers' questions into Spanish for Jessie, as she was not completely comfortable with English. At home Jessie's dad spoke Spanish and Estelle said she also spoke Spanish at home, but not as well as him. The sister who came on the first visit was 4 years old at the time, and she preferred to hold up four fingers rather than saying the word, four. Jessie's older sister was 12 years old.

Prior to the first study visit, Estelle said her daughter "feels neutral about math." Jessie had been learning about addition and subtraction at school, in part by playing math games with blocks, but at the time of the initial visit she had not yet been introduced to multiplication and X-Y graphs. Estelle was very open about her own feeling about math: She said she didn't like it. However, Estelle was very excited about the prospect of participating in this study. She said, "I was very honored that our principal asked us if we wanted to be part of the math exhibit." Estelle took lots of photos during the interim visits and readily shared them with the researcher.

The family had a museum membership and had been coming to the museum about five times a year, and Jessie had also been to the museum on school field trips. Jessie was shy at times when talking with the researchers and museum staff, but her mom was easy to talk with and forthcoming. During some interviews Jessie gave very short answers. Her mom usually expanded on her answers, sometimes answering the questions on her own when Jessie had little to say.

This family made five study visits to *Math Moves!* plus several non-study visits as a family or with school classes (Table 3.2). In addition, Jessie and her mom told us they had visited *Math Moves!* several times before the first study visit. The study visits lasted from about an hour (for the first visit) to an hour and a half (for the interim and final visits).

Study Visit	Length of visit in mins.	Who visited	Description of Visit
#1	65	Jessie, Estelle, and Jessie's younger sister	Initial study visit. Estelle introduced the exhibits to Jessie, explaining how to the use them and explaining what happened.
#2	90	Jessie, Estelle, Jessie's younger and older sisters, Jessie's father	Estelle stepped back a bit and let Jessie's father and older sister help Jessie at the exhibits.
#3	90	Jessie, Estelle, both sisters, Jessie's aunt	Estelle let her daughters engage with the exhibits more-or-less on their own. Jessie showed off the exhibits to her aunt, who elicited explanations by asking Jessie questions.
#4	90	Jessie, Estelle, both sisters	Estelle had prepared a writing assignment for Jessie to complete at the exhibits. Jessie's sisters spent the visit elsewhere in the museum.
#5	90	Jessie, Estelle	Final study visit. Estelle and Jessie used the exhibits together as a museum employee taped the visit and occasionally engaged with Jessie and her mom.
Note: This family also made numerous non-study visits to the exhibition: prior to the study starting, between the initial and second study visits, and between the third and fourth study visits.			

Table 3.2: All five study visits for this family lasted about an hour and a half. Estelle worked closely with Jessie during all the visits, sometimes with help from Jessie's older sister. Jessie got a chance to demonstrate what she knew about the exhibits to her father and aunt on two of the intermediate visits, and to a museum employee on the final visit.

During the initial study visit Jessie and her mom interacted very closely, with Estelle working hard to make sure her daughter focused on the exhibits and took something away from her experiences. During the initial visit both Jessie and her younger sister stuck really close to their mom, which Estelle said was pretty typical of their visits to the museum and to other places—and even at home the girls tended to stick close to her. During this first visit Estelle made use of what she had figured out about the exhibits during her pre-study visits to *Math Moves!* She took the lead at most exhibits, explaining what each exhibit was about and helping Jessie engage with it, often in ways suggested by the labels. Jessie mostly paid attention to her mom and did what she suggested, although towards the end of the initial visit Jessie insisted on doing things her own way at the two exhibits about shadows—*Moving Shadows* and *Shadow Table*. At *Moving Shadows*, mother and daughter both tried to control the sliding light source. Then, at *Shadow Table*, Estelle stood back and let Jessie take the lead, although she still stepped in a few times to offer advice or help when Jessie encountered difficulties or did things in a way that varied from what Estelle seemed to consider the right way to do things.

Figure 3.14: During the initial study visit Estelle worked very closely with Jessie at most exhibits, including Pattern Generator (left photo), showing her what to do and explaining the exhibits to her. However, towards the end of the visit, Jessie took over control at the shadow exhibits, with her younger sister playing alongside her (right photo).

For the second study visit Estelle had help from Jessie's older sister and father in supporting the younger kids' use of the exhibits. Perhaps because of this, Estelle hung back more, taking photos and, at least at times, letting other family members help Jessie when she needed it. For instance, Jessie's older sister helped explain the *Lever* exhibit, which Jessie had trouble figuring out. Jessie also set her own challenges at some exhibits, for instance building a "pyramid" at *Blocks*. Overall, Estelle spent much less time in teaching mode during that visit.

Figure 3.15: During the second study visit Estelle stepped back a bit and let Jessie's older sister help her at Lever and other exhibits (left). Jessie came up with her own challenge at *Blocks* (right).

During study visit three, Jessie's aunt came along. Estelle said Jessie was excited to show her aunt the exhibits, and that Jessie gave more explanations about the exhibits than usual, partly in answer to her aunt's questions. Otherwise the children mostly engaged with the exhibits on their own, and the adults stood back and made suggestions, some of which were followed and some not. Estelle implied that, for visits two and three, her family behaved as they would have if they were not in the project—mostly having fun at the exhibits, skipping a few exhibits that they didn't like as much, showing off for a relative, giving some attention to labels and challenges, but without much teaching behavior on the mom's part.

That all changed for the fourth study visit. Although both sisters were along, the older sister took the younger one elsewhere in the museum, leaving Estelle and Jessie on their own in *Math Moves!*. When asked what was different about this visit, Estelle said, "There was more learning this time." Because she was concerned that Jessie was not learning at the exhibits, Estelle had developed questions about seven of the exhibits for Jessie to answer in writing. She explained that some questions were inspired by the exhibit labels, and some she came up with on her own. Jessie wrote her answers in a notebook during and just after her engagement at that exhibit. Here are examples from two exhibits, first *Pattern Generator*, and then *Balance*.

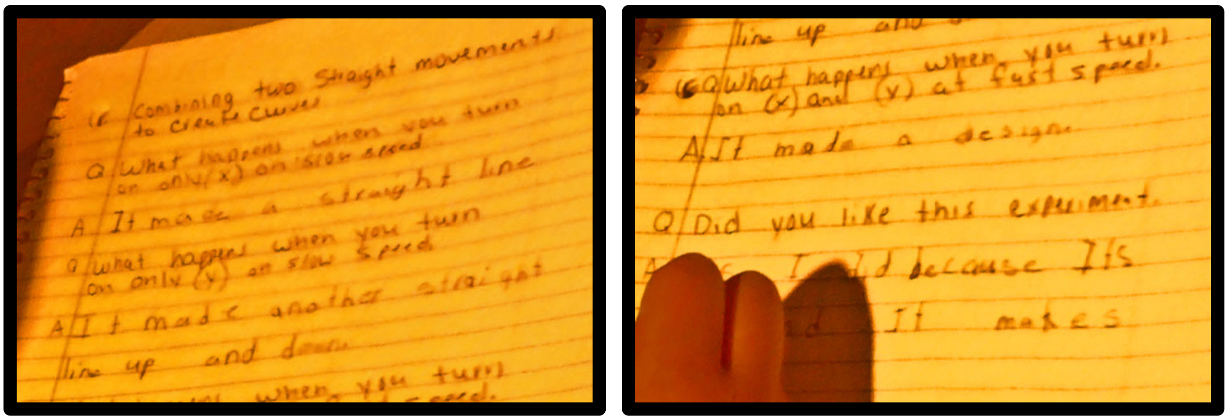


Figure 3.16: During the fourth study visit Estelle prepared questions for Jessie to answer at *Pattern Generator*. "Combining two straight movements to create curves" is written at the top of one of the labels for this exhibit.

Combining two straight movements to create curves. [*Pattern Generator*]
Q What happens when you turn on only (x) on slow speed.
A I made a straight line.
Q What happens when you turn on only (y) on slow speed.
A It made another straight line up and down.
Q What happens when you turn on (x) and (y) on fast speed.
A It made a design.
Q Did you like this experiment.
A Yes I did because its [text obscured]. It makes [text obscured].

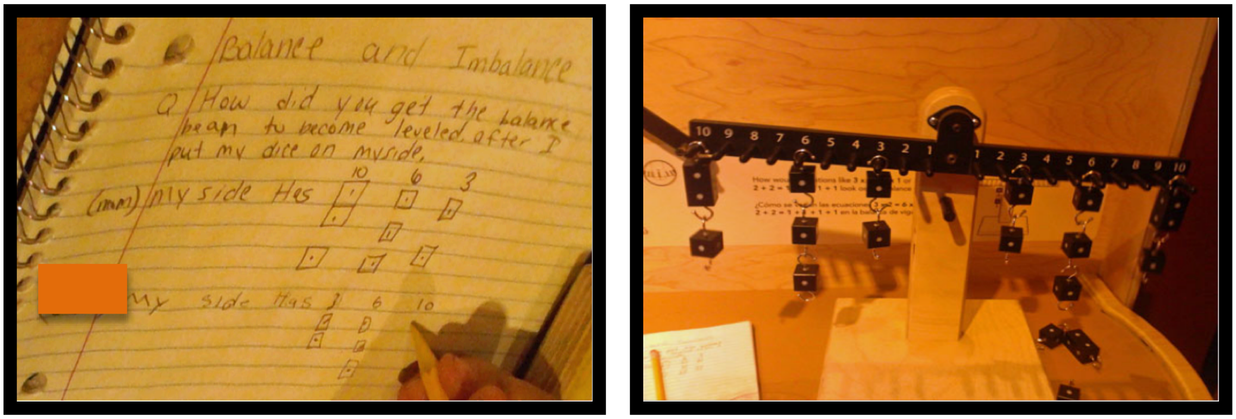


Figure 3.17: During the fourth study visit Estelle prepared questions for Jessie to answer at *Balance*. Jessie's answer was a mirror copy of her mom's dice.

Balance and Imbalance [*Balance*]

Q	How did you get the balance beam to become leveled after I put my dice on my side.			
Mom	My side has	10	6	3
Jessie	My side has	3	6	10

Estelle said she came up with this idea on her own, because she wanted Jessie to feel she was actually learning something at the exhibits rather than "just playing." The mom said that, with this approach, her daughter spent more time at each exhibit, rather than quickly "skipping" to the next one. She also said that because Jessie wrote and asked questions about the exhibits she understood more about them. Asked if her daughter needed coaching to complete the notebook pages, the mom said they worked together to write down and answer the questions. During the follow-up phone interview for this visit, Jessie often turned to the notebook to answer the researcher's questions.

Figure 3.18: During the fourth study visit Estelle had Jessie write answers to questions about seven of the exhibits in a notebook.

During the final study visit Estelle once again took charge of the visit, deciding where to go and what to do when they were there. Jessie sometimes objected, but almost always gave in at the end, even at the last few exhibits, when she seemed tired. Estelle was very much in a teacher role during this last visit: Setting tasks and challenges; giving directions, and then stepping in and doing it herself when Jessie did not follow her directions; asking questions, and then answering the questions herself when Jessie did not answer or did not answer the way Estelle wanted her to; and explaining things that were happening at the exhibits as she understood them. Jessie seemed too worn out to talk much during the final interview, but Estelle had lots to say about that visit and her family's overall experience with *Math Moves!*

Figure 3.19: During the final study visit Estelle played a very active role, operating and explaining many of the exhibits even after Jessie was tiring.

Over the course of the project this group made some use of label and screen challenges at the *Math Moves!* exhibits, but both mother and daughter came up with their own challenges as well, which were sometimes related to the label challenges and sometimes not. For example, during the initial study visit they attempted one of the exhibit's illustrated challenges at *Rainbow*, and they did a screen challenge at *Sliders*, but at *Blocks* and *Drawing Shapes* Estelle came up with her own challenges that may have been inspired by the labels, but were somewhat different.

During the third study visit Jessie's aunt tried to get her started with the video challenge at *Blocks*, but during the interview Jessie said it was boring. As described below, she preferred to build her giant pyramids of blocks, a challenge that her mom made a bit harder during the fourth visit by suggesting she try removing a block without having the whole structure collapse. During the final study visit they attempted a mix of challenges that were included on the labels, and challenges that they came up with on their own. For instance, at *Blocks*, Jessie built a pyramid structure first, then her mom tried to get her to build what was shown in the video. However, they got distracted and never really talked about the concept of doubling. At *Rainbow* Estelle read the on-screen challenges at the tops of some graph screens, and they tried to make the drawings, with some success. At *Sliders*, Estelle tried to get Jessie to do the circle challenge, and when she would not, Estelle took over for a bit and drew parts of the circle, carefully tracing the line. They mostly came up with their own balancing challenges at *Balance*, although at one point Jessie tried to copy the weight setup illustrated in a label (which was not really intended as a challenge).

Blocks was Jessie's favorite *Math Moves!* exhibit, and she returned to it during every visit. Her favorite activity most of the time was building a pyramid-like structure out of the blocks. She tried a doubling challenge once or twice but either abandoned it or explained that "it was boring." During the third study visit she made a stair-step structure. When Jessie would build a pyramid or stair-step, she was engaged with a number of math skills including spatial reasoning, pattern recognition, and engineering, but her engagement in ratio and proportion was minimal.

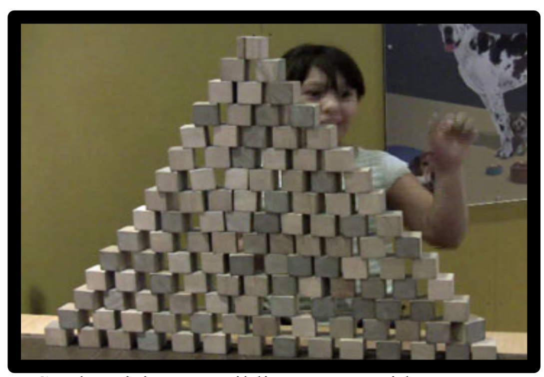
Study Visit 1: Copying a structure.


Non-study Visit: Building a pyramid.


Study Visit 2: Building a pyramid.


Study Visit 3: Building stair steps.


Study Visit 4: Building a pyramid and removing a block to see if it would collapse. It didn't.


Study Visit 5 – Building a pyramid.

Working through these exhibit and more personal challenges, Jessie and Estelle developed their fluency with using the exhibits during the course of the study, and with some of the math skills that could be applied at the exhibits such as counting, addition, and subtraction. They also evolved their kinesthetic fluency to some extent. There was less increase in fluency in recognizing quantitative relationships, and with using numbers.

Because of their pre-study visits to *Math Moves!*, by the time of the initial study visit Estelle—and to a lesser extent Jessie—were already somewhat fluent with the mechanics of many of the exhibits. Jessie gradually increased her kinesthetic fluency particularly at *Rainbow* (drawing with her body) and at *Sliders* (drawing with her arms) during the interim visits, and by the later study visits Jessie seemed to be getting more fluent at controlling shadow sizes at the *Shadow Table* and knowing where to put the next weight to balance the scales at *Balance*, even though she wasn't able to articulate her understandings using words. At *Clicker* Jessie and Estelle learned to slide a wheel out from the center to make the clicks faster, or to use a smaller wheel to achieve the same effect.

Although they sometimes used and referred to the numbers at the exhibits (e.g., *Clicker* and *Balance*), Estelle and Jessie did not increase their fluency with using numbers or understanding quantitative relationships very much at most exhibits. For example, at *Clicker* they noticed the rail position numbers, but used them exclusively as reference points. At *Blocks*, the use of numbers was limited to primarily counting, with no comparing before and after counts when enlarging, and there was no discussion of doubling.

By the time of the final study visit Estelle seemed rather fluent with operating most of the exhibits, although she sometimes misunderstood various things about what the exhibit mechanisms actually did and how they worked. On this last visit it was hard to judge what Jessie's fluency at operating the exhibits would have been like on her own, since her mom directed their engagements. Estelle often understood the qualitative relationships and at least some aspects of the numbers and quantitative relationships embedded in the exhibits more than her daughter did, but not always. And sometimes Estelle only partially understood some of the relationships and math, while Jessie was closer to being on the right track. At many exhibits, Estelle passed her incomplete or incorrect understandings on to Jessie, even when Jessie was initially on a more accurate path towards understanding the exhibit.

Neither mother nor daughter noticed or paid attention to most of the numerical ratios and proportions that were embedded in many of the exhibit experiences. One of the reasons for this seemed to be that the exhibit graphics didn't provide much support or guidance for parents with limited understanding of math, like Jessie's mom. To work for someone like Estelle, the labels would need to provide clear and simple explanations that could be understood "on the fly" and also model how to talk about the exhibits with their children. Such explanations may have been buried in the minute-long phone messages at some exhibits, but they were not provided by the labels and screens.

During the course of the study, Estelle found several ways to connect the exhibits to her and her family's lives. During the initial study visit, she briefly pointed at and named the clock and musical notes on the *Clicker* photo montage, although she didn't say much about them. At

Spirograph, Estelle shared her memories of the "spiralgraph" toy she loved when she was young. During later visits both mother and daughter connected the shadows at *Moving Shadows* to sun-made shadows, and the moving light to the mobile light source used at their dentist's office. Both *Moving Shadows* and *Shadow Table* exhibits reminded Estelle of kindergartners discovering their shadows at school. Other school-based relationships that Estelle mentioned included *Sliders* reminding her of drawing on graph paper in math class; the "X and the Y" of *Pattern Generator* also reminding her of school math. Jessie had not yet studied graphs at school, and she did not use much arithmetic at the exhibits, which may explain why the exhibits did not remind her of the math she was doing at school.

Jessie's feelings about her *Math Moves!* experiences seemed to vary a lot during the study, although her mom seemed to maintain a very positive attitude towards the exhibition through the entire year and a half. For instance, Estelle said, "Every time we come [to the museum], that's where we always start. Even on our non-visits, when we just come, we always will start at the math exhibit."

During the initial study visit Jessie's reactions to the exhibits seemed mostly positive and enthusiastic—she seemed to be having fun. Neither Jessie nor her mom seemed bored or tired of the exhibits, even though this was not their first visit. Because Estelle had already figured out things to do at most of the exhibits, Jessie rarely got frustrated by the exhibit mechanisms. However, Jessie seemed a bit frustrated with her mom's close supervision at times, and she wanted less attention from Estelle towards the end of the visit. Estelle also seemed to get a bit frustrated during the initial visit when her daughter did not do things in ways she considered proper, and she often stepped in to set things right.

Visits two and three seemed to be more relaxed and pretty fun for both mother and child. It was interesting how Jessie showed a sense of ownership of the math exhibits during these visits and enjoyed showing off the exhibits and her use of them to her father and aunt. Although Estelle seemed very satisfied with the notebook assignment she had her daughter complete during the fourth study visit, it was hard to say how Jessie felt, since her mom was present for the entire interview. During the final study visit, it seemed like Estelle was having a great time and did not want it to end. Jessie seemed to have fun for the first 40 minutes or so, but then started to tire and become increasingly resistive to trying new exhibits. If she was also getting irritated with her mom, she covered this fairly well from Estelle; however, sometimes she put her head down on an exhibit or looked at the video camera with a sad face.

Despite her pre-study visits to the exhibition, Estelle said that she had not known this was a math exhibition until the researchers walked them there for the initial study visit. During the final interview Estelle talked about whether this felt like a math exhibit to her yet. She said, "It doesn't feel like math to me….I think because I find the exhibits fun. So I think that's why it doesn't feel like math." She continued, "It's not like we're adding and multiplying to solve our exhibit." However, she said about the *Balance* scales, "I did feel like math, because we had to count how many we used." When asked if it was a good thing or a bad thing that the exhibits did not feel like math, Estelle said, “I think it’s a good thing, because I’m not very good at math. So, I find it fun.”

Asked what was the best part of being part of the research project during the final interview, Estelle said, “That we all get to work on it together. And we seem to learn something new every time we came. Because there were a few [exhibits] I didn’t know how to do when I would just come. But our visits, we would just ask someone, ‘How does it work?’” When Estelle was asked about the most challenging part of being a research family, she said, “Getting Jessie to do the exhibits and actually learning something from it. Because she was like, ‘Oh no, let’s go to this one.’ And I’m like, ‘You have to take your time to learn how to do one exhibit first, before you walk away from it. Not just give up on it.’” So, as noted above, Estelle took her responsibilities as a recruited respondent very seriously, especially during the last two study visits.

In some of the other recruited groups, some parents who had stronger backgrounds in math engaged in more of the quantitative aspects of the exhibits with their children, including doing the doubling activities, and using the numbers embedded in the exhibits. While this vignette shows how a family group that does not have a very strong math background can still successfully engage with and enjoy the *Math Moves!* exhibits, it also demonstrates a visitor group that, in spite of all the effort that Estelle put into working with Jessie at the exhibits, appeared to plateau in their fluency. This is not necessarily a negative outcome—many children need to repeatedly practice, and engage in repetitive behaviors—but it begs the question of whether with different labels, support, or guidance she might also have been able to begin to engage with ratio and proportion in meaningful ways.

3.6 When Things Go Awry

Sometimes at the *Math Moves!* exhibits, things didn't go quite as planned or hoped. This section briefly outlines some of these issues. Each of these issues strongly influenced visitors' experiences, often undermining their use and understanding of math at the exhibits, and sometimes contributing to the development of negative feelings about and attitudes toward math. While some of these issues have already been dealt with, it will be important for the remainder to also be addressed.

Some exhibit units were particularly challenging for visitors because of maintenance and/or related design issues. For example, *Clicker* tended to be particularly sensitive to visitor use. Problems RRs reported included the clicking mechanism on the wheel sometimes sticking, the wheels not turning when placed on the disk, and the covering on the disk bubbling and tearing. Also, even when visitors set up two wheels so they clicked at the same rate, the clicks often drifted out of sync pretty quickly, apparently because of friction effects somewhere in the system. Thus visitors who seemed to be on the verge of discovering the role that proportions could play in meeting the label challenges, instead decided that they had once again failed to meet those challenges. One young respondent actually did figure it out when she put the 9" wheel on the 18, explaining that 18 was double 9, and then placed the 6" wheel on 12. But the clicker on the 6" wheel was sticking and not clicking each rotation so she assumed she was incorrect, tried something else which also didn't work, and eventually gave up (D6-6). Many RRs expressed repeated frustration when this exhibit in particular was not working correctly on multiple visits. And some RRs didn't realize when it was not working and thought they weren't able to be successful.

Photograph courtesy of B1

Figure 3.20: The covering on the disk was sometimes bubbled and/or torn which made the exhibit work incorrectly.

A common error made by some respondents was when they misinterpreted the numbers on the *Clicker* wheels as fractions; they actually are wheel diameters in inches.

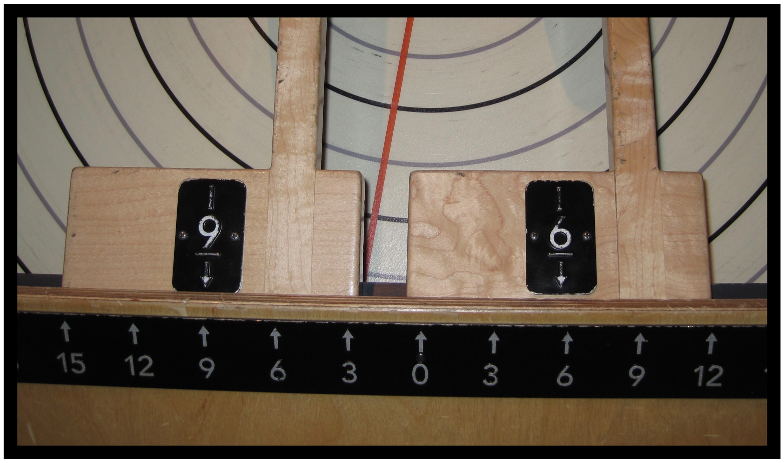


Figure 3.21: When respondents mis-read the numbers on the paddles as fractions rather than the diameter of the wheel, they were unable to figure out the quantitative relationships at *Clicker*. In other cases, respondents read the numbers as 19 and 16 respectively, which similarly prevented them from increasing their quantitative fluency. Both of these led to feelings of frustration and inadequacy.

An example of this comes from the written debrief of a telephone interview with a child who had just completed his fourth study visit to the exhibit.

> [Child] explained that there were numbers…underneath the exhibit and that there were fractions on the wheel like 1/6 and 1/3 and he spoke about making the fractions equal to make them click at the same time. When asked to explain further he said "I don't really know where to put the 1/6 and the 1/3 to make it click, but I'm thinking you'd put the 1/3 at the 2 and the 1/6 at the 1." (D2-4)

Another respondent, one with a strong math background, also struggled trying to make sense of this exhibit, ultimately giving up in frustration during his fifth and final study visit.

> [Child] said he's been trying to figure out what the numbers on the bottom mean. He said he thought it had something to do with the numbers on the wheels themselves. He read the numbers on the wheels as "19" and "16". He knew that in order to get them to click at the same time he had to have both wheels at the same "start" position in relationship to the actual clicker….He then said that maybe he could ignore the "1" on the wheel making it "3" and "6" instead of 13 and 16. Then he went back to saying "13 and 16" and tried putting the "13 wheel" on the number strip 13 and the "16 wheel" on the number strip 16. (D4-5)

When respondents made these sorts of misinterpretations, it was virtually impossible for them to discover the role that proportions play in meeting the label challenges, and often led to frustration, and feelings of inadequacy.

A few respondents talked in detail about some of their frustrations with trying to get the wheels at *Clicker* to click at the same time, and ultimately figure out the quantitative relationships.

> [The adult] said he thinks [*Clicker*] is too "wide open. It doesn't validate anything. It gets frustrating and we leave, which is probably why we didn't do it today." He continued, "I didn't get ratios in High School, and in a year and a half here, I'm not going to get it from that exhibit." He said he really likes the idea [of the exhibit], and knows he should be able to get it to click at the same time, "but I can't do it." (B5-6)

Another RR figured out how to get the wheels to click at the same time, but was not able to test his hypothesis until the wheels were fixed.

> [The focus child] was able to get two wheels to click at the same time [for the first time]….[The adult] said previously they usually gave up because "I wasn't quite sure what I should do." [The child] described how he did it: "I took two of the ones labeled one sixth and put each of them on an 18 on the bottom." [The adult] asked: "Like a denominator? What do you mean on the bottom?" He explained that he was using the number strip under the disk "So I think that's how you get them to go at the same time. You have like one sixth to 18, and one sixth to 18, or one third to 18, and one third to 18, or one third to 9 and one third to 9." [The adult] asked: "So did you base everything on 9?" [The child] said "No, I just put them there" (meaning that he just somewhat randomly picked 18, but purposefully put both wheels on 18. [They] talked about the fact that those were the same size wheels. [The child] went on to say, "Yeah but I'm also thinking that if you put the one third wheel on the 9, you could make it go at the same time as the one sixth on the 18. But I couldn't test that because the one third wheel was sticking…because one third is double one sixth, and 18 is double 9. He agreed that next time he might check it out presuming that the clickers are fixed. He said the clickers on the one third and one twelfth wheel were both sticking—that it would "click and then just stays there." (B1-4)

The *Shadow Table* exhibit resulted in missed opportunities with some RRs because the magnet with the retractable cord was sometimes missing, or it didn't retract. This resulted in only a few RRs using it. A few other respondents tried to use it, but it sagged, and didn't create a straight line when the magnet was placed against the screen, causing visitors to become confused about its purpose, and ultimately unable to engage with that numerical concept.

Sometimes the scales at *Balance* were just slightly off. Some visitors were able to understand that they were "close enough," but many others struggled trying to figure out how to make, for instance, the ratio at the Ratio Scale exactly 1:2 (Figure 3.22), ultimately giving up in frustration and not able to figure out where their math mistake was.

Figure 3.22: When the Ratio Scale sometimes became slightly off, some respondents were not able to recognize that this was “close enough,” and that their weights really did represent a 1:2 ratio.

Additional issues have already been discussed in other sections of the report. They include issues such as the lines on the graph at *Rainbow* dropping down whenever a visitor stepped off the main path; visitors misinterpreting the exhibit because they didn’t understand the science or mechanics behind how the exhibit worked, for example at *Rainbow* thinking you make a graph by jumping onto a pressure sensitive mat; critical aspects of the exhibits (especially aspects related to numbers) being ignored, misinterpreted, or not noticed, such as the numbers on *Clicker* and *Shadow Table*, and the ratios on the *Spirograph* wheels.

Some visitors got sidetracked in ways that were counter-productive to engaging with the qualitative and quantitative relationships in meaningful ways. One (particularly tall) RR father had been trying since the group’s initial visit, to make a line as smooth as the ones shown on the drawing challenges. All the lines that the family members were able to make were lines that included small peaks and valleys (Figure 3.23) and weren’t as straight as in the label. The group was not able to figure it out until the final study visit when they happened to bring a shorter family member along, and discovered that this smaller person made the smoothest lines of all. With help from the researcher during their final study visit, this family group finally realized that the sensor was positioned at such a height that it picked up *leg* movements in the tall people, movements that were relatively jerky compared to the *body* movements of the youngest daughter. This group was limited in how far they were able to increase their kinesthetic and qualitative fluency at this exhibit because they got sidetracked trying to figure this out, and spent a limited time exploring other challenges.

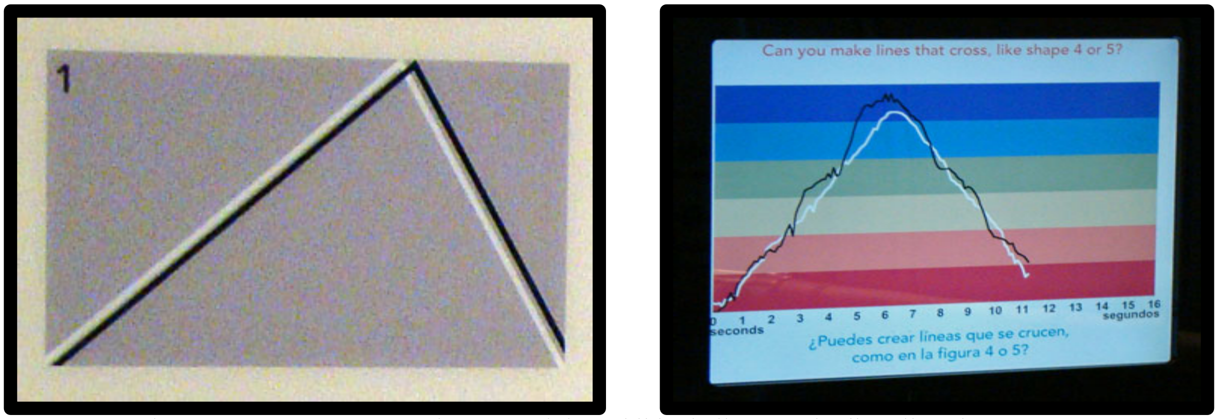


Figure 3.23: One RR group interpreted the exhibit challenge quite literally. They became frustrated when they thought they were unable to reproduce the drawing because they weren't making as smooth a line as on the label.

One deep-thinking respondent noticed what he identified as a "flaw" in *Sliders*.

> [The focus child] described how he made diagonal lines [at *Sliders*] that were labeled 2:1 and 1:2 on the screen by moving one arm faster than another. "This is cool; it's telling you what kind of line you are making." He then went on to explain ratios and x,y coordinates very clearly. He moves on to stair steps and explains unison. Then he discovers [what he believes to be] a mistake on the exhibit program. He says the two diagonal lines are mislabeled: the 1:2 line should be 2:1, and the 2:1 line should be 1:2. He says that in school he's learning this, and that x is always listed before y. He then went to another screen [one demonstrating octave ratio] to confirm his statement. He then gets [the other members of his family] and says, "Guys, I found a flaw!" He then goes on to explain the mistake on the screen to them. (B1-6)

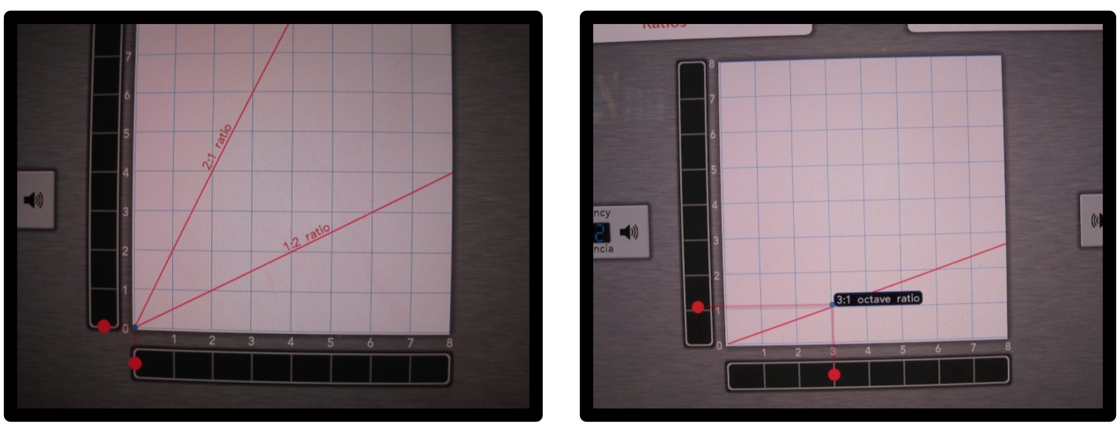


Figure 3.24: One RR focus child identified a "flaw" in the *Sliders* program on the graph on the left, i.e. that all graphs give the x coordinate first, followed by the y. He then went on to confirm his hypothesis by checking it against the graph on the right. He concluded that he was correct and that the exhibit did indeed contain an error.

According to a project team member, the graphs are technically accurate, the problem stemming from seemingly contradictory notations. In other words, the graph on the left depicts a *slope* whereas the graph on the right depicts a *point*. In the case of the graph on the left (referring to a slope), the upper line (labeled “2:1 ratio”) has a change in y of 2 for every change in x of 1. And vice versa for the lower line labeled “1:2 ratio”. The graph on the right (referring to a point, and labeled as “3:1 octave ratio”) is—as the respondent correctly points out—depicted as the value of x first followed by the value of y. Labeling the two lines on the graph on the left as “2:1 slope ratio,” and “1:2 slope ratio” respectively, would cue the visitor that these two graphs are depicting different types of ratios.

Another source of frequent frustration and confusion with some respondents was a difficult-to-resolve glitch at the *Rainbow* exhibit. In spite of the project team’s best efforts, when visitors veered off the edges of the mat, the line being drawn on the screen would suddenly drop out or drop to the bottom. (Figure 3.25). Many respondents began to use the exhibit, and just when they thought they were starting to understand how it worked, the lines would drop.

To some respondents the dropping lines seemed to happen at random intervals, but many others were convinced that it was because they had done something wrong. They persisted in trying to correct their mistake, but in most cases, either were unable to do so, or they came up with an (incorrect) answer that made sense to them, but often proved counter-productive to them having a meaningful (or enjoyable) math experience.

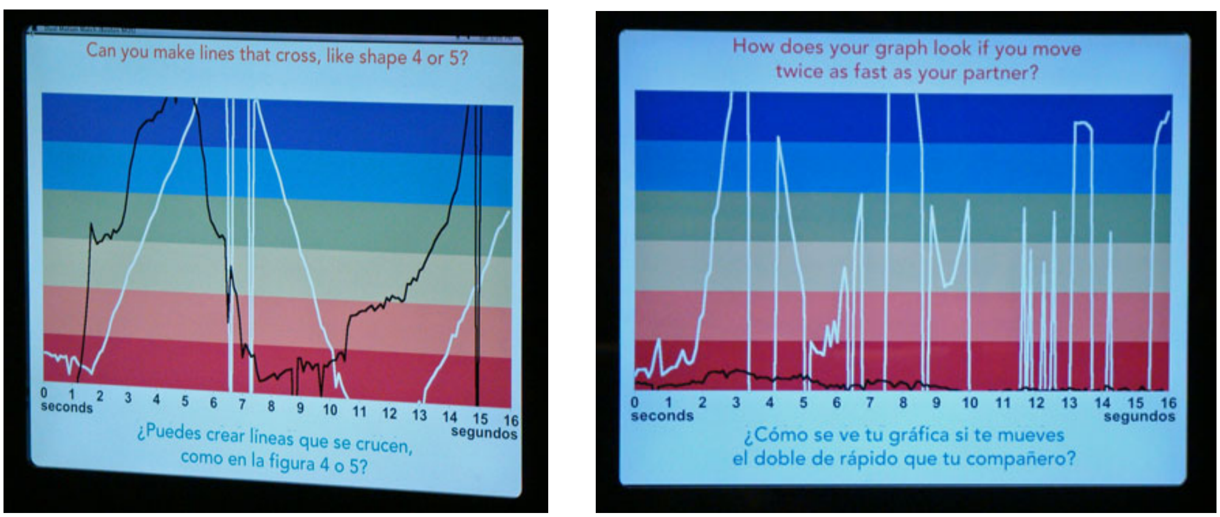


Figure 3.25: The lines on the graph at *Rainbow* would suddenly drop to the bottom of the graph any time a visitor’s foot strayed off the main path, often sidetracking visitors and making it difficult to use the exhibit, and difficult for respondents to increase their kinesthetic fluency.

3.7 Participating in this Project

Comparing Recruited and Casual Respondent Experiences

All RRs knew they were part of the study, and also knew they would be participating in an interview following each study visit. It was not usual for example, during the initial study visit, for respondent children (and adults) to glance often at the video camera, and sometimes fumble with their microphones (which sometimes came loose). In addition, the recruited groups were confined to the *Math Moves!* exhibits by our protocol, whereas casual groups more likely wandered in and out, visiting other exhibits and then wandering back for another math exhibit or two. In addition, when the researchers were on site, at times they approached a recruited respondent and talked with them about what they were doing, and in at least one case, staff kept an eye on a young sibling of one respondent, who would not stay near his mom and sister.

Note that a major difference between these RR families and casual visitors is that these were *cued* respondents. They knew we wanted them to experience these as math exhibits, and that changed their behavior in many ways, some of which they recognized and probably some that they did not. Because of our interviews, recruited groups—especially older children and parents—were probably thinking more about the math in these exhibits than most visitors, and also were thinking about the entire experience—at least to some extent—as a math experience. (There were indications that casual visitors may not even think of *Math Moves!* as a coherent math exhibition.)

On their first visits, recruited groups spent more time overall in *Math Moves!* than during their later visits. During these initial visits, they tended to spend more time at each individual exhibit, and they also tended to see more exhibits than during subsequent visits. Observations and interviews suggested that the recruited parents also stuck much closer to the focus children and spent more time in teacher mode than was observed with most casual groups. Adults also seemed to read more labels and read them earlier in the engagement than casual visitors did. As one mom put it, the biggest difference from their usual museum visits was that she did more directing to get her son to focus and think about the exhibits. She explained that usually she lets him wander from exhibit to exhibit, and he will spend little time at most of them, but then find some where he sits and spends some time, but then wander from exhibit to exhibit again. On the *Math Moves!* study visits, he would focus a lot on one exhibit, and then focus on a second exhibit, and then on the next one, and so on. She explained that it was exhausting for him. In summary, this mom said that the *Math Moves!* exhibits were more structured than their usual visits. (A5-6)

Unobtrusive observations of casual respondents revealed many children in the 6-12 years age range exploring the *Math Moves!* exhibits on their own, with parents sometimes sitting on a couch, looking at a cell phone, spending time with preschool siblings, and so forth. When the researcher was able to follow several casual groups through most of their visits through *Math Moves!*, it what was striking that they stopped at relatively few of the exhibits before moving on to another, often adjacent exhibition. Then they sometimes returned to a few more *Math Moves!* exhibits, and sometimes not. In other words, the casual experiences with *Math Moves!* seemed pretty fragmented and incomplete compared with the way the recruited respondents used the exhibition.

In summary then, on their first visits the recruited respondents tended to behave like "ideal" visitors. If the exhibits and their interpretation didn't "work" for the recruited families, then they seem unlikely to work for casual visits. But even if the exhibits did work for recruited families, there could still be problems—because most visitors are not "ideal." The recruited groups' interim visits seemed more like casual visits—i.e. less time spent overall, fewer exhibits stopped at, and less engaged parents. It appeared that perhaps these interim visits provided a more realistic view of how the exhibits work with more typical, casual visitors. The final visits, of course, were controlled in large part by the researchers, so they were even less like casual visits.

What Was It Like Being Part of a Research Study?

After the final study visits, all RRs were asked what it was like being part of the research study. All the parents and children said they thoroughly enjoyed the experience and would recommend it to friends. Some however, said they would tell their friends to consider carefully the time demands before they decide.

As noted earlier, these respondents could name many ways in which their *Math Moves!* visits differed from their typical visits to the rest of the museum: They usually came to *Math Moves!* first, spent more time and effort in the math exhibits, made sure the kids stopped at all of the exhibits, made sure the kids "learned something," and so forth. It turned out that, for many families, these differences were what made their *Math Moves!* experiences special.

Parents said they liked spending more time focusing on their children's exhibit experiences, and they liked seeing the resulting gains in understanding. As one mom said, the best part of the project was "that we all get to work on it together. And we seemed to learn something new every time we came." (A4-6). Another mom said the best thing about being part of the research was seeing her son gain confidence. She said that he is often more of an observer than a participant. By participating in *Math Moves!*, he has been a participant and gained confidence. She explained that seeing that happen was "quite satisfying" to her. (A2-6).

A dad said it was rewarding to come here and do educational things that they normally wouldn't do with school work. "This is something totally outside of the regular homework and reading and stuff that we do at home. So it was fun to actually see how they progressed just in terms of how they learn. Watching how they learn and how they absorb things as they've gotten older in the last two years. So that's been kind of fun." This dad also said that it was fun just to spend time with his children and to be able to come to the museum. "I loved coming to the science museum as a kid, so it was fun for me to get them involved to come down here." (C1-6).

One mom focused a bit on her own experience, saying she enjoyed being able to get a better understanding of what the exhibits were all about, because she came to see them time and again.

She also said she enjoyed seeing her son get a better understanding of the exhibits. (A5-6) One dad put a slightly different spin on things. He said that being part of the research forced them to be more deliberate about their interaction with the exhibit. He said, because he knew he was going to be interviewed, "that made it more enriching, knowing that I have a responsibility. I really need to try to learn something or at least have some questions." (C2-6).

And some focus children appreciated their parents' attention, especially when it came at the expense of their younger siblings. One mom explained "When I'm here, I can find a little bit of time that I don't have to be with all my other children. When I'm home, I always have stuff to do….With [the focus child], we never have time to be by ourselves. Now that we're doing this, we have a little time." Her daughter agreed, saying she had not gotten "to be alone with her mom for about 7 or 8 years" (A1-6). In a different family group, the focus child said he was pleased to have time alone with his mom at the exhibits, with his younger brothers at home or elsewhere in the museum with his grandmother (A5-6).

Several of the recruited respondents talked about how being a part of the research project made them feel special. One focus child talked about how it was fun to be videotaped during the first and last visit (A5-6). A mom explained, "I was very honored that our principal asked us if we wanted to be part of the math exhibit." She thanked us for inviting her family to be part of the study (A4-6). Another mom said it was great to be part of this, not just to help "the museums be the best they can be," but "also to teach our children responsibility." (C1-6).

A few respondents mentioned the perks they received as research participants. One focus child said the best part of being in the study was that she got to come to the museum more often. Her mother agreed that she really looks forward to coming, so a big perk was getting the membership (C3-6). Another mom said one of the best things for her was being able to come during spring break and bring cousins, aunts, and uncles. They would not have been able to do that without the membership (C5-6). Other respondents focused on the food, especially the hotdogs for a younger brother (C2-6).

A few of the families talked in positive terms about the math they had learned by being part of the study. One focus child said that because she learned ratios and fractions here, she would probably understand it more at school now. Her mom said that what her daughter was saying was what she was hoping to hear when she signed up for the project, since both she and her daughter have struggled with math. She also said that she liked that they were thinking about math in a non-traditional sense (C2-6). A mom said, "Having to spend more time with the math exhibit has been good. Because [her son has] struggled in math, for sure….I think it's helped him. And maybe if we didn't know we were going to be answering questions afterwards we might not have spent quite so much time" (said with a smile) (C5-6).

Without exception, recruited respondents took their participation in the study very seriously. One set of RR parents explained that they had told their children that going to the museum and going to all the *Math Moves!* exhibits was their job. The mom explained that they had enrolled in this study in part "to teach our children responsibility." She continued, "[The children] bragged to everyone that they had a job at the science museum." The dad added, "They were very proud of that." (C1-6).

One young respondent who was one those who increased her fluency the most, explained that she really liked being a participant, getting free food, talking on Skype with the researcher, and playing with the exhibits. She said her favorite part was the free food. When asked what wasn't as much fun she said "maybe it was coming back to the exhibit over and over again. It was cool the first couple of times, and it was cool the whole time, and after a while I kind of knew what was coming." She said she got that feeling around the third or fourth time….She said she liked having people listen to her opinions and knew that what she was doing really mattered and hoped that she would be "immortalized." (D6-6).

Another respondent also spoke favorably of being in the study, but also thought that it was too many visits, explaining that their child had started to lose interest. The adult said that it was great that the sixth visit was cancelled because the child expressed resistance beginning with the third visit, saying "We've seen it." (D4-5).

Scheduling study visits was often a challenge for RRs because it was important that the timing not only fit the respondent's schedule, but that the interviewer would also be available within a few days after the visit. And of course there were other challenges as well. One mom said the most challenging thing for her was "getting [my daughter] to do the exhibits and actually learning something from it. Because she was like, 'Oh no, let's go to this one.' And I'm like, 'You have to take your time to learn how to do one exhibit first, before you walk away from it. Not just give up on it.'" (A4-6). Another mom said something similar. The most challenging part for her was that she wanted her daughter to spend a certain amount of time in the math exhibits, and the mom felt she had to try to explain things to her daughter more than elsewhere in the museum (C3-6). And one mom remembered the trouble she had e-mailing the pictures she took with her cell phone during the interim visits (C5-6).

One dad talked about the most challenging part for him was when they came on a weekday when there were lots of school kids of field trips, rather than on their usual early Saturday morning time, when the museum had fewer visitors. His wife said she gets the impression that [their children] think of this as "their exhibit. How dare somebody be at my station?...Anywhere else it doesn't matter, but it's sort of become their part." Her husband said there were no times when the children resisted coming to the math exhibits. "They loved coming." (C1-6).

What Was the Experience of Professionals?

A secondary research question for this evaluation focused on the experience of the 30 museum (and related) professionals who were part of the project (18 core team members and 12 advisors), and how their participation contributed to their thinking about (a) the development of math-related STEM visitor experiences; (b) the development of ISE opportunities for repeat visitors; (c) their own understandings of and relationships with concepts related to math, ratio, and proportion; and (d) the power and efficacy of collaborative projects.

This section of the report will explore these and a few related issues. As noted previously, the findings for this section of the report came primarily from nine depth interviews conducted with purposively selected *museum professional staff*.[13]

Embodied Cognition

As part of the interviews, respondents talked in depth about exploring ideas related to embodied cognition and the important role that the research had in their thinking about math and developing math exhibits.

> My biggest takeaway [from this project] was related to the work that Ricardo Nemirovsky is doing about the embodied knowledge of these concepts [such as proportion]…. Movement, and using space, and describing things in gestures. That was a new idea for me, and something where I now see it happening. (SC1)

> I guess one thing that was really great for me, because I'm not a mathematician, and in some way the ideas that we get very early on that insist that math is really about numbers and manipulating numbers. I would say that Ricardo and Molly and Tracey, having those guys as part of this core team, they were really great in being able to affirm that math is ideas and a way of experiencing the world, and there is a value in that…I think it gave me a deeper appreciation for the potential for learning in exhibits, even if a visitor doesn't parrot back the messages we write down…As opposed to work with this for 10 minutes and then take a bubble test. And see if you can do this equation. These exhibits are working in a different way than how we work at math when we're in school, and there's value in that….I had that affirmed in this project, especially with the participation of Ricardo, Molly, and Tracey. Because they live their lives doing this, thinking about how people learn math, and what will help them be stronger learners of math. And they see these kinds of experiences as beneficial. And they're able to point at things and say why and what's going on in a way that I can't. And I think that's great. That was very affirming to me, and helpful. (SC2)

> The researchers, I think, really helped shape the project in their participation, because they had such strong ideas and experiences with how people learn, and the effectiveness of videotaping people and attending to gestures as a form of communication, to get us thinking outside of some of our normal thought patterns. (SC9)

[13] Although they were referred to as museum professional staff, in fact some of the *Math Core* advisors were not museum professionals, but rather math educators and/or researchers.

> [The respondent] talked about Ricardo's presentations on embodied cognition at the Albuquerque meeting. They [described] an exploration where they moved their hands while facing the walls and following his prompts to embody ideas like half and twice, working with partners. [The respondent] said they frequently came back to that experience as they worked through exhibit development. [The respondent] recalled that as the time when they really started to talk about large-scale, full-bodied experiences and said "That really shaped the development and led to things like, or led to the emphasis and energy behind things like the big shadow wall and the big chairs, and *Partner Motion* [*Rainbow*], and the dance math one that didn't make the cut. And even the smaller scale ones, like the Theremin [*Sliders*] and balancing [*Balance*], all involving you moving these parts, and you having a sense of what that felt like, and the scale of that. An important thing about the project is that realization. [The respondent] talked about Ricardo and Molly sharing videos of an exhibit…where kids were gesturing and that "that was the way they were kind of processing their math learning. Even if they didn't have the right words for it they would use their gestures for it. That was a big thing I've learned." (SC3)

> I'm really interested in that, and I actually think there's a lot museums can do there. I don't feel like I have quite the grasp on that that I would like. My grasp on mathematically thinking is still a little tenuous for my sense of confidence about it. And embodied cognition also is a little bit less, even less solid in my brain. But it's so interesting. You know, we talk about creating experiences for visitors that maybe they don't have the sort of mathematical sophistication or comprehension to really understand now, but that we hope, we have this vague idea, that when they actually then get to studying a concept like this in school they'll have these experiences to draw on and say, "Oh, that's like when I was at the museum, and I did this thing." And that they have those memories to pull forward and then sort of mathematize or whatever else [if it's a] science concept. And I feel like embodied cognition is another way to think about that nonverbal, nonschooled, nonformal learning that we might be able to help visitors construct. (SC7)

Other respondents described how they continued to wrestle with concepts related to embodied cognition as a learning phenomenon.

> It's just a very different kind of learning than I'm accustomed to. So, as I say, I just don't have a good handle on it….I keep looking for someone to explain it better and I haven't come across it. So it just may be incommensurable with my way of thinking. But I suspect not. (SC5)

And another respondent referred to the Albuquerque presentation where the project team did "full-bodied ratio and proportion, sensing it kind of on a human scale," but explained that they didn't find it particularly applicable to their own work.

> It was interesting. It was a little bit out there when they were doing it. At the same time, you know, I guess the exhibit that's most closely tied to some of the stuff they were showing that ended up in *Math Core* was probably *Partner Motion* [*Rainbow*]. I don't know. I thought it was good. I don't know how directly it ended up applying to some of the exhibits. But it was a good kind of angle to have approaching some of these exhibits. (SC6)

Content vs. Experience

Many respondents had a lot to say about discussions throughout the project related to how explicit to make the math. It was unclear the extent to which these discussions were resolved, or the extent to which they helped museum staff professionals evolve or change their thinking; but it was clear that respondents were thinking about the issue.

> And I think the other thing—like is it the experience or how closely this adheres to the math?—I don't think it was intended to be resolved. But it was intended to be brought out so that people could all see things a little bit more broadly and start finding ways to reconcile those even if we couldn't do it at that point in the exhibit. (SC1)

Some respondents explained that the exhibits are successful experiences, even if visitors aren't talking about math.

> Would every person on that exit interview talk about math? Especially if the exhibit weren't called *Math Moves!*? I mean, I don't know. But, I guess I believe that you can kind of get it without knowing you're getting it." [The respondent gave the example of a visitor moving the rabbit closer to the light, showing it getting bigger, but not quantifying that, and then walking away and not calling it math because it doesn't look like school math.] "I don't think that makes it unsuccessful. I think that makes it maybe even more successful. So that was not something that concerned me, or our team here….We can make the experience good, let them walk away with an enjoyable experience, and if the math is embedded and inherent in it, they'll eventually pick up on that and be OK with that. (SC3)

Another respondent echoed the need for museums to provide pleasurable experiences, and not be too heavy-handed with the math.

> I would lean more on the side of the experiencing rather than communicating. Kids get way too much of the communicating end. And I think that museums have an important role in providing pleasure. And this communicating isn't always pleasurable. And again these multiple entry points have to allow for the little girl to get pleasure out of making sounds as well as eventually, maybe in a couple years she comes back and engages with the X-Y space. (SC8)

This respondent felt strongly that one of the jobs of the *Math Core* project was to help change cultural preconceptions of math.

> Part of what the *Math Core* team is grappling with is we are trying to push our cultural perceptions of what math is. So, the emphasis on open-ended design, the emphasis on bodily experience, the emphasis on multiple ways of engaging. A lot of these things are trying to offer an alternative vision of what the discipline of mathematics is. So that's part of the work on the one hand. On the other hand is all this kind of accountability pressure. And everybody is bringing a whole set of life histories of very schooled expectations about what mathematics is. And so I think that combination of things creates what on a good day is a really productive tension [laughs], but maybe on a bad day creates a sort of feeling of frustration. [The respondent explained that the question of visitor experience vs. content learning is already beginning from a] rigid and schooled notion of what content is. (SC4)

This next respondent however, felt a need for more explicit math.

> The respondent described that it would have been nice if there was more emphasis on engaging visitors with the mathematical content.] But it seems to me that the exhibits really shied away from that. A part of that is that there's very little text. And that was a decision that the team made. I don't know that there's a place that defines what a ratio is, or what proportional reasoning is….[And at *Rainbow*], what's the mathematics there? What am I really doing? Or what's the mathematics of the table-top shadow exhibit….You at least provide that kind of information. [The respondent expressed concern for the people who just come for 20 minutes.] Are they getting any math? (SC5)

But another respondent had a different perspective.

> We're fearful of the content argument part of that. "Well, if they can't satisfy an answer on a test, did they really learn anything?" I tend to say yes. They're experiencing the stuff, and whether they recognize that it's specifically ratio or proportion or math, or whether or not there are numbers attached, I still think there's value there. Then there's the other end, where I always feel like they're just trying to satisfy some sort of evaluation component. (SC6)

As can be seen from the many ways that respondents talked about the content vs. visitor experience issue, this issue was not something that everyone agreed on. Respondents indicated that the discussions themselves were a valued part of the process, even though, in the end, their thinking wound up in different places.

Thinking About the Exhibit Development Process

Respondents indicated that for the most part, the exhibit development process within each museum did not evolve all that much as a result of participating in this project.

> There wasn't really any uber developer or uber manager who got to say, "Ooo, this looks like a good idea, why don't you work on that." Each museum developed from their own internal processes exhibit ideas that they thought met the terms of what was laid out in the grant and brought those to the whole group for consideration. [The respondent described J. as] very generous [at letting each museum work the way they work.] There wasn't really too much assigning; it was more staying in contact and each museum kind of benefitting from the input that they got from their colleagues at the other museums. (SC2)

> [The exhibit-prototype process] was nice in that each of us could work within our own strengths on the process and then come together and get feedback. So I thought it worked really well that way. I've never been in such a strong collaboration that way, where you can each work within your own strengths yet still be working to meet others' needs. (SC9)

The one area that respondents talked about that did make a big difference in how they thought about designing exhibits, was when they were able to see first-hand observations of visitors using the exhibits, and especially the role the social group of which they are a part plays.

> I think for myself personally the thing that had the most impact on me was observing the interactions between parents and children, or adults and children….I have very vivid memories of ways in which these dyads, adult-child, interacting with the exhibits, could either facilitate or sometimes hinder the children's interactions with the exhibits….I took a lot out of that and it has made me think differently about how potentially powerful but also how difficult the work of [an exhibit developer or] museum facilitator might be because of the other adults that the child is accompanied by. [The respondent gave an example of a girl, about 5-6 years, who was using *Sliders* to make sounds, not looking at the screen. Her father tried to get her to focus on the screen, her mother rolled her eyes, and the girl just went back to doing what she had been doing. The respondent talked about] the need to develop objects and spaces where there are multiple entry points into the activity and not a single necessary outcome. [The respondent explained that what the girl was doing was a fine entry point, and that the dad might have actually been hindering her engagement.] [There is a need for] low threshold entry points. (SC8)

> When we put things out, we think that we are planning exhibits. But we really are not the planners of exhibits. What the visitor decides to do with the exhibit. While I do believe that we can create the conditions that increase the chances that somebody is more likely to do this than that. I think the idea of affordances, the features of the materials, what they afford or dis-afford or mis-afford, are really such firm messages, non verbal, physical, tactile kind of messages, and visual messages. That part, in a way, becomes…what our intention is for the exhibit. [The respondent talked about 4 yr old and 8 yr old girls who walked up to *Blocks* and said] "You want to build a house?" Those blocks have such an overwhelming message about build-and-stack…. It really struck me how little we take into account about what people are really going to do with things. Because we're in love with our intentions. (SC1)

Developing Exhibits for Repeat Visitors

The idea of designing exhibits for repeat visitors over time, was a new (and appreciated) concept for some respondents.

> [This respondent explained that the idea of] learning over time [was really new. The respondent described it as trying to develop an exhibit that has a sense of being] iconic [i.e. that people will want to come back to it over and over again, and then use in different ways as they get older.] (SC7)

> [The respondent talked about] making these be experiences that people could use over and over again and kind of learn through time. And trying to make experiences that people would want to do over and over again, I think to me that relates to how people learn math. Like I've noticed families who have, the first time they play with the balance rods, for example, they aren't ever using numbers to talk about why is it balancing. Or maybe an older kid in the family is using numbers to describe the same concept that a younger kid is using just words to describe…It takes time to build those concepts into ways that you can articulate them. All these exhibits are really great because you can experience the concept even if you don't have the words for it….I know that was a goal of the project, and I actually see it happening, so that's cool. (SC3)

Other respondents said they considered this to be the way they always developed exhibits.

Understanding Ratio and Proportion

Some staff respondents talked about how their own understanding of ratio and proportion evolved, specifically because of their participation in the *Math Core* project.

> The concepts of ratio and proportions were internalized a little bit more. I don't know if it's new, but that they're not interchangeable. I have kind of a rudimentary understanding of some of these math concepts. To practice with them was very useful. (SC1)

> I think I have a deeper understanding of ratio and proportion. I think I have some real curiosity about how ratios and proportions work. I'm always really fascinated by…multiplicative proportionality, and how, when you talk to your kid, well I just remember thinking about this when I was a kid. "When I am 20, [someone else is] going to be 40, so I'm half her age….It convoluted me. "How can that be? How can that be?" [The respondent talked about paying more attention and talking more about ratios and proportions with the family as a result of this project.] [I show more to my kid, and] I've become more aware I think. (SC2)

> I keep telling people [about how my ideas about math have changed] and I feel like such an idiot that I'm so old to come to this realization. While we were working on the project I realized that when something is within my human scale there's nothing abstract about math. And it seems so simple but I just didn't know it before….[The respondent clarified it was about things you could measure with your body or grab with your hands] as opposed to the train down the track….I just think of it more personally. I have a more personal relationship with math now, a more personal understanding. (SC9)

> I hadn't really articulated for myself that a proportion is a ratio of ratios….I certainly hadn't thought about. We use "half" in so many ways, no wonder it's confusing to people….It was really interesting to me to think about how challenging that is for someone who isn't interested in just jumping right in. (SC7)

But, like many other staff respondents, this respondent did not completely separate personal understandings of this topic from how visitors think and learn about such things. For instance, the respondent talked about finding out about multiplicative reasoning as a type of arithmetic reasoning, and about some of the development aspects of multiplication.

> One of the great things about this profession is, you're always learning something about how visitors understand stuff, or how to present things, or what makes something interesting….How do you talk about fractions to a 6 year old? Well, you start with half, because half is one [fraction] that is much easier for them to get a grasp on. How do you convey, and what are the very very very many meanings of half? (SC7)

Another respondent also talked about how participating in the project helped with understanding more about designing exhibits and tasks for children to engage with math.

> It was emphasized [in the discussions] how children will do with an object you design whatever it is they want to do. Whether your goal is, you think, clearly laid out or not….So, yes in terms of how to design tasks that are low threshold and where children can engage in different ways [my thinking has evolved.] (SC8)

Another respondent talked about a personal process for developing exhibits about math, starting with a richer understanding of the topic of ratio and proportions:

> [My increased understanding of ratio and proportion was gradual, like] the exhibit's about math, ratio, and proportion. How do you get that to be interesting or engaging? [You have discussions about what ratios and proportions are, and you break it down further.] I'll go back to the most basic example. If somebody has got a balance beam, and they've got uniform blocks, and they're trying to balance that beam with various numbers, at its core, that's a ratio issue, or a proportion. Two of these on this side, to one of those on that side. [So if you break it down to that base level there are a lot of] end points for coming up with something engaging. [Once you break down a concept like ratios and proportions] there are a lot of ripe avenues that you can take to produce an exhibit that addresses the subject matter. (SC6)

And, as described above, lots of respondents also talked about what they had learned about what math is and math learning from the researchers at San Diego State. The researchers' ideas about embodied learning, especially as they related to ratio-related ideas like half and double, did seem to change how several of the team members thought about math and math learning.

This respondent explained that participation in the project helped with understanding better how museum professionals (and others) think about math and math learning, especially about how deeply entrenched cultural expectations are for what math is.

> [For] people coming in with schooled expectations, it's harder to validate their experiences as being genuinely mathematical. [There are] widely circulating notions in our culture about what math is. And so we're all sort of various[ly] subjected to those. Visitors certainly, the *Math Core* team is sort of [as well] influenced by that, but also in the act of resisting those ideas. (SC4)

It was interesting to note that participants in the *Math Core* project did learn things about ratio and proportion and evolved their own fluencies, but also learned about how children and others learn about that topic.

Participating in the Collaboration
Staff respondents were overwhelmingly positive about participating in the *Math Core* collaboration.

> I have very positive views of the collaboration….From my perspective, it looked like it worked really really well….[The exhibits] have a similar kind of feel of this very open-ended, exploratory kind of thing. They're not heavy handed about do this, do that. But it's like, here's some things you can play with and perhaps discover something about ratios as you're engaging with them. (SC5)

I felt like the collaboration was really, really, great, and I think that the kind of overall how it worked was that we had these three key meetings in person…and those were the times that the team got to be together, see things hands on, talk face to face. In between those times each team would be advancing their four prototypes on their own with like weekly check-ins with the team, showing pictures, talking about it, and that was really fruitful feedback to take into account all the other institutions experiences and expertise into our process to make your process wider and more informed than you normally can make it when it's just your institution. So I felt like it made all of the exhibits better in general but also better in the thinking that they would have to work in all four places. So we weren't just giving feedback on colleagues, we were giving feedback on an exhibit we knew was going to eventually live at our place, too….So there were some compromises that had to be made that like, something that would work really well at [one institution] in the way that they had originally prototyped, it might not work that well at some of the other institutions. And so we gave feedback for ways to kind of meet in the middle. And I thought it was a really great process. (SC3)

[Working with another collaborative project is] giving me an appreciation for how, sort of, well-structured and organized J. has made the collaboration among the museums. I think, my overall sense is that the collaboration has been really productive, sort of striking. And I think part of it has to do with the sort of synergistic philosophy that Paul at Explora, J. at the Science Museum of Minnesota, Ricardo, Troy at Museum of Life and Science. I think that there's a lot of agreement at the level of, sort of, philosophy of pedagogy. And I think that kind of created a real community around this. (SC4)

It was really kind of a treat to have this one big project to direct our focus to for quite a long period of time….I think it worked out really well. I think that going into it our apprehensions might have been, "Hey, you know, we've got a style of exhibit and exhibit criteria that we try to stick to for the museum." And I think there was apprehension, you know, "Hey, will we be able to get those kinds of exhibits to be accepted within the larger group of *Math Core*, and will we end up with exhibits on our floor that meet the same standards that we have for the others?" And I think we did. So I think that was the biggest concern, and it ended up being all right. (SC6)

I think there's just tremendous value in figuring out how to work cross-institutionally. I think we were able to accomplish some things that were really fabulous that none of the four of us would have done on our own. That our exhibit is richer and more interesting than anyone of us would have [done], given the same set of parameters. And I don't think that any one of the four of us would have picked those parameters exactly, either. So I really feel we accomplished a really cool thing that we couldn't have otherwise….We improved on each other. We pushed each other to do more, better. (SC7)

Many respondents attributed the collaboration's success to its overall design and project management. "J. is a master with collaboration." (SC9). Other respondents expressed similar sentiments.

> [It] was more complex and elaborate [than other collaborations I've been part of], and actually really well managed. Because there was the exhibits part, and then there was the research that Ricardo was doing; and they were just sort of interweaved every once in awhile. And then there were the four museums, and a big set of advisors. And it wasn't sort of a formula in advance for what each museum would do: "Do six, nominate two, recreate three." It [emerged] from kind of brokering priorities around the variety of the math experiences, the quality of the exhibit experience, the contribution that each of the museums were bringing to the whole project. And how it had to work at the different museums. There was a lot there that was complex, and I think it was really well managed. Well managed by not managing it too closely….[It was] really complex. I would not want to manage something that complex….I think it worked out really well. I think all of the partners are still in the project. I don't mean to be flip, but I think there are a lot of projects where there are some competing interests or the pressures of different organizations or the relationships are…challenged….Everybody is still active in the project, and I think that shouldn't be underestimated as an accomplishment. (SC1)

> [The] project design was really, really well thought out. [J.] was really wise [at selecting the partner institutions] somehow. There was a collegiality that developed working together. That each of the four institutions was excellent in its own ways, and that somehow the sum of the four was greater than each of the parts. I think that part of the project plan was masterful. Because we were all working together, it caused us to stay really focused on the stated project goals. And that was a real strength. I don't think there was a lot of creep away from the project goals….I think we focused on the goals, and I think we met the goals….I'm very proud of the work, and I enjoyed this project immensely. (SC2)

The well-planned and relatively frequent meetings (both face-to-face and telephone conferences) were highly valued by many respondents.

> Well-used, thoughtful regular phone conversation helped us work really well together as a whole team and come to common understandings of what we were trying to accomplish. (SC7)

> [The respondent described being a fan of the face-to-face meetings, and that the three big meetings were important.] In terms of just project team dynamics and stuff like that, to have a face-to-face meeting really early on, and then basically every six to nine months to have a face-to-face meeting, I think really helped with getting to know everybody, so that you knew everybody's voice on the phone and felt comfortable writing e-mails to people. That was important. I've also been on projects that you maybe go further into the project work before you have that first meeting….The energy around the collaboration in-person is always stronger. So it was important to have those meetings….[And the value of the phone meetings was increased by] really regularly sending out minutes. Sending updates and summaries and schedules was a strength of the project, also. (SC3)

The diversity of museums was frequently mentioned as an important contributing factor to the success of the collaborative.

> The diversity worked really well….To bring those different perspectives all into one room—you know, kind of a wisdom-of-crowds kind of thing—I think was a real strength of the project….I think it's interesting because these four museums, we all decided to work together because we all believe in similar things. Even if our institutions look different, I think we have a lot of the same values….If we think of us as four museums on a spectrum of slightly more traditional to more transactive, I think this project helped us all kind of maybe slide along that spectrum to a place where our museum might not have gone without the collaboration. (SC3)

> [The] institutions at play here are really different….And I think that there were maybe times when it was a little hard to figure out how to deal with different styles of doing development, and just doing prototypes, different aesthetics, you know, like how hardened an exhibit should be, how finished it should look. So I think that maybe it's a struggle with any cross-institutional collaboration. There were definitely moments when I think that J. sort of struggled with how to deal with that….I think [the struggle paid off]. I think that *Math Moves!*, as an exhibition, does have a level of coherence, but at the same time I can kind of see the voice of each of the institutions coming through. (SC4)

> [The collaboration was] very unique [because] each of these four institutions have quite a different visitor profile. [Each institution was] very particular with certain conditions and certain strengths that they brought to this. And we all had to work pretty hard to make our respective exhibits work for the other three partners. (SC2)

In addition to the interesting mix of museums, respondents also mentioned the importance of the diverse group of project staff and advisors, a group that included exhibit developers, mathematics researchers, university professors, evaluators, and advisors from many walks of life.

> There was a feeling that this was a shared endeavor that [was] made better by the input and thoughtfulness from this large group of people that were invested in the basic idea. (SC1)
>
> [We all shared] a total passion of education, and for sharing math, and figuring out how to engage visitors, and trying to create experiences that are meaningful and fun. (SC7)
>
> We were pushed to a place that is not necessarily where we would have gone left to our own devices. Now, as a developer, I thought that was great. I thought that was a very positive influence on the work here. (SC2)

Some respondents mentioned a few things about the collaboration that could have gone smoother. One respondent felt they could have been more fully involved.

> One thing I would say is that, other than those three [in person] meetings, I haven't really heard much about the project. So I feel a little bit disconnected. Keep us in touch maybe. I know I should have done a better job of logging into the Wiki space that the project had, and I didn't do that. But if I had received e-mails every six months just to update, I would have appreciated that….Without knowing what the constraints of budget and time, etc., I think that my expertise could have been better utilized. (SC8)

Some respondents mentioned a desire for more coordination between the research and evaluation. After seeing video clips of from both the research and evaluation, one respondent explained that they would have appreciated an opportunity to discuss some of the videos. "I do think that it would be worthwhile to have a much more, sort of, extended exchange about this stuff." (SC4) One respondent said that it would have been "really powerful" to have a video posted, with a virtual space for people to log in and ask questions or write their comments. (SC8)

The development of the graphics and labels appeared to be one of the more challenging aspects of the collaborative for some respondents. One respondent pointed out that the process felt fast-tracked with limited time for input, although another respondent felt that the process took too long. Another pointed out that there was no time to test most of the labels with visitors and then revise them. Someone else pointed out the challenges of reconciling the four museums' very different philosophies about what goes on a label.

Finally, a few respondents mentioned that technology was not used as much as it might have been to further the work of the collaboration. They mentioned the Ning and Wiki, which they felt had the potential to be a good collaborative tool, but that neither seemed to be used that much, at least partially because the user interface was not particularly user-friendly.

Contributions of Math Core

In addition to the many opportunities for rich and meaningful professional/personal development and networking, respondents talked at length about the many contributions of *Math Core* to the field of informal science education. These included pushing envelopes—about the meaning of math, about the types of exhibits museums can (and should) do, about how visitors learn, about exhibit and exhibition development. Contributions also included lessons learned about collaborations and what they can accomplish, and how they can (and should) be managed.

4. WHAT WE'VE LEARNED SO FAR

About the Overall Exhibitions

Overall it seemed that most visitors found fun things to do in *Math Moves!*, and all of the recruited respondents (and at least a few of the casual visitors) found challenges to meet by using math and related skills. Some exhibits, such as *Spirograph* and *Rainbow* remained very popular with the recruited respondents even after six visits, and were listed as favorites by many focus children. However, there were indications that some of the exhibits did not keep many children and their adults engaged over multiple visits (like *Triangles* and, for many respondents, *Chairs*). No respondent said that *Math Moves!* was their favorite exhibition in the museum, but it seemed to be about in the middle of the pack, and most RRs had a few individual *Math Moves!* exhibits they really liked.

Part way through the study, there were indications that some families were losing interest in the exhibition as a whole, but that interest was revived somewhat when adults provided new challenges, or researchers provided hints about how to meet the challenges.

If you think about math in the broadest sense—including qualitative and kinesthetic relationships—it was clear that *Math Moves!* seems to be engaging visitors in many different mathematical ways, from making basic qualitative comparisons between variables, to (somewhat less frequently) using basic math skills like measuring and graphing, to (much less frequently) using numerical ratios and proportions to meet challenges set by the exhibits or by others in their groups. Because 6 and 7 year olds participated in this study, the ratio-related operations that could be understood by multiplication (like doubling) were happening more frequently than the ones that used or looked like division.

About Individual Exhibits

A large amount of data about individual exhibit units was amassed as part of this study, but these data are beyond the scope of this report because of the focus on the visitor experience over time, and the limited resources available. The data collected includes understanding what mathematical outcomes visitors are getting from each exhibit and why/how they got them, plus evaluation of what worked and what didn't work at each exhibit.

About Visitors Learning and Using Math in Museums

As was described above, all the recruited respondents—children and adults—learned at least something about math as they used the exhibits. Most respondents' use of the exhibits got more sophisticated over the life of the study, and some older children in particular evolved at least some of their use of some of the exhibits to highly sophisticated ways. Some of the younger children tended to plateau in their use of the exhibits, remaining at fairly basic engagements especially with the numerical and quantitative aspects. But it's uncertain how much of this numerical math younger children would have done if their parents had not taken on the teacher role. Most young respondents needed support and guidance in order to discover and use the quantitative tools and, if they got that far, to understand the roles that ratios and proportions played at the exhibits.

While all recruited respondents engaged in increasingly sophisticated ways with math over the course of the study and practiced many math skills, the (limited) data gathered about the casual visitor experience indicated that most casual visitors' experiences with ratio and proportion will likely be limited to doubling and halving. Even most of the RR adults needed help discovering ratios and proportions at exhibits like *Clicker* and *Spirograph*. A few highly motivated visitors may still recognize and understand the more complex aspects of math at these exhibits, but even motivated visitors could benefit from more guidance and support. In some cases attentive floor staff could provide this, but ideally, with a little tweaking much of this can be achieved with revised labels that are designed to help visitors navigate through the more common bottlenecks.

There were some indications that with revision of the challenges and labels in particular, more numerical ratio and proportion opportunities at the exhibits could become accessible to more visitors. Note that we are not advocating for more elaborative or explanatory labels. Rather we are suggesting labels that are strategically designed to jump-start meaningful visitor conversations, and to provide the minimum amount of guidance or hints necessary to get visitors to focus on the most important—but often overlooked—aspects of the exhibit. Labels such as this require extensive rapid prototyping with intact family and other social groups so that they can be carefully tweaked and re-tweaked until they achieve the goal of catalyzing meaningful age-appropriate conversations about ratio and proportion (Perry, 2012, p. 11-37).

Although beyond the scope of this study, it is also likely that for some respondents, embodied (Nemirovsky, Kelton, & Rhodehamel, 2013) or visceral learning (Perry, 2002) may have taken place and contributed to their understandings of math, ratio, and proportion in ways that this study was not designed to assess.

About Designing and Conducting a Longitudinal Multi-Site Evaluation Study

This type of longitudinal evaluation study focusing on a small number of respondents over a relatively long period of time is an immeasurably useful way to look at the ways visitors engage with museum exhibits. In many ways, this study was more similar to a research study than an evaluation study. A very large amount of descriptive data was gathered, but unfortunately resources were limited so much of it was unable to be reported. For example the study took more than twice as many researcher days as was budgeted, in spite of the fact that all respondents didn't complete all of the six study visits originally envisioned, and even though we weren't able to analyze some of the data in as much depth as would have been desirable.

About Designing Exhibits for Repeat Visits

While developing museum exhibits that appeal to repeat visitors is part of a number of museums' exhibit development philosophies, deliberately designing exhibits for repeat visitorship is a relatively new challenge for most museums. In conducting this evaluation, we tried to tease out some of the underlying characteristics of what makes for an exhibit that is successful with repeat visitors. In this project, it started with a list of desired exhibit characteristics (see Table 1.1):

- *open-ended*, enduring, unlimited, and unexpected; to encompass several ways visitors may interact with the exhibit, and often more than one (related) math problem to explore.
- *conversational,* inviting and supporting parental engagement; to encourage children and their parents to talk with each other about the exhibit activity
- *parental engagement* is supported; materials carefully prepared to help parents and caregivers become exploratory learners side-by-side with their children
- *transactive*; archaeology that shows physical evidence of prior use
- *accessible*, and multi-sensory for everyone, by incorporating physical and cognitive universal design techniques and audio and written labels in English and Spanish
- *kinesthetic*, supporting and facilitating whole body and sensory learning

Some of the characteristics listed above turned out to be particularly important for repeat visitors, but the data revealed additional insights. For example, one exhibit developer talked about one aspect of open-endedness, citing the importance of multiple variables and multiple challenges, such as *Balance* having three different scales, so you can do one during one visit, and a different one on another visit. This respondent also mentioned *Rainbow* as having six different challenges, and that they escalate in terms of difficulty and skill required, so visitors can work towards drawing the elephant over several visits. *Shadow Table* has lots of different parts to play with, and having multiple measuring devices at *Chairs* gives visitors something different to try the next time they come. (SC3).

Those exhibit qualities did prove important, but it was really more than just multiplicity that made these exhibits work. One of the reasons that these four exhibits (*Balance*, *Rainbow*, *Shadow Table*, and *Chairs*) were as effective as they were, was not simply that they had multiple challenges or ways for visitors to interact, but rather each had a variety of appropriate ways for kids of various ages/stages of mathematical development to engage in productive and satisfying ways, and for adults of various levels of sophistication and math ability to find ways to support and facilitate their children's interactions, even when those ways included parents and children learning together, side by side.

Some recruited respondents also commented about how having lots of parts and lots of challenges might keep them coming back for more. One RR child described in some detail why he would probably continue to come back to both *Spirograph* and *Rainbow*, and what might be helpful for keeping him engaged at other exhibits.

> [The focus child] said he would NOT get bored by *Spirograph*, because he hadn't tried all the gears yet or worked much on the side with two gears. He also said he could do *Rainbow* more times without getting bored because he would try to "get, like, the elephant." He would want to do more with *Big Shadows* if there were different challenges posted….He said he [probably wouldn't get bored if the museum would] switch out the challenges and add more gears [at *Spirograph*]…. He also agreed that having more blocks at *Blocks* would be good, and [his mother] suggested adding some different shapes there. He also agreed that adding some more shapes at *Shadow Table* would be good. (C5-6)

Another aspect of the exhibits was that because they were designed to incorporate kinesthetic and some whole body opportunities, at certain exhibits visitors could literally grow into them, for example at *Chairs*, and also at *Sliders* when children's arms had grown long enough by the final study visit where they could work both sliders at once.

Another exhibit developer talked about how a three year old might not get anything out of a hypothetical exhibit except that if you turn this crank something will happen. They explained that if the child came back at age five or six they're going to build on that. "So the idea of having multiple entry points sort of speaks to that. And then the *Math Core* exhibits, some of them are fairly dense. So I could see a person coming back over time and still finding out more about it. Or even just, through their understanding or improved dexterity on something like [*Sliders*], just getting better at it. Or giving themselves more challenging problems." (SC6). Similar to young visitors physically growing into exhibits as described above, this fits fairly well with this study's finding and observations of recruited respondents' intellectual changing how they used and talked about the exhibits—for example, the youngest respondents incorporating their developing understanding of multiplication into their engagements at *Blocks* and *Chairs*.

A staff respondent described another aspect of designing for repeat visitorship as "trying to make experiences that people would want to do over and over again" (SC3). As this respondent explained, "I think to me that relates to how people learn math. Like I've noticed families who have, the first time they play with the balance rods, for example, they aren't ever using numbers to talk about why is it balancing. And maybe an older kid in the family is using numbers to describe the same concept that a younger kid is using just words to describe.…It takes time to build those concepts into ways that you can articulate them. All these exhibits are really great because you can experience the concept even if you don't have the words for it." (Or in some cases, the numbers for it.) This same respondent also talked about *Rainbow* as an example. At this exhibit, young visitors can draw the diagonal line, but not know to use the term slope to describe it. The respondent said "I know that was a goal of the project, and I actually see it happening, so that's cool." Again, data from this study confirmed that this was happening with most of the recruited respondents.

As noted previously, parental engagement proved to be a critically important aspect of the most successful and satisfying visitor experiences. Parents also played important roles in keeping young respondents coming back for more, because they pointed out the multiple parts and multiple challenges that kept things interesting. As described elsewhere in this report, older children in particular responded well when the sophistication of the challenges matched their developing mathematical abilities, but they often needed parental support to both notice and work their way through the more complex or sophisticated challenges available at some of the exhibits. Adequately supporting and facilitating this parental engagement at this level was a big challenge for most the *Math Moves!* exhibits. Many RR families talked about the desire to have more hints, appropriate challenges, read-at-a-glance interpretation, and clues that would direct their attention to the numbers in the exhibit, but without overwhelming them or contributing to feelings of inadequacy. Additional research that explored this important aspect of designing exhibits on math (and other topics) for repeat visitors is something that would greatly benefit the museum community at large.

REFERENCES

Allen, S., Gutwill, J., Perry, D. L., Garibay, C., Ellenbogen, K. M., Heimlich, J. E., et al. (2007). Research in museums: Coping with complexity. In J. H. Falk, L. D. Dierking & S. Foutz (Eds.), *In principle, in practice: Museums as learning institutions* (pp. 229-245). Lanham, MD: AltaMira Press.

Ansbacher, T. (2013). Making sense of experience: A model for meaning-making. *Exhibitionist, 32*(1), 16-19.

Ansbacher, T., Hein, G., McLean, K., Rounds, J., & Spock, M. (2000, Fall). Meaning making: The conversation continues. *Exhibitionist, 19*(2), 38-47.

Borun, M., & Dritsas, J. (1997). Developing family-friendly exhibits. *Curator, 40*(3), 178-196.

Csikszentmihalyi, M., & Hermanson, K. (1995, May/June). Intrinsic motivation in museums: What makes visitors want to learn? *Museum News, 74*(3), 34-37, 59-60, 62.

Diamond, J. (1986). The behavior of family groups in science museums. *Curator, 29*(2), 139-154.

Falk, J. H., Scott, C., Dierking, L., & Rennie, L. (2004). Interactives and visitor learning. *Curator, 47*(2), 171-198.

Garibay Group. (2013). *Design Zone Exhibition summative evaluation*. Unpublished manuscript, Oregon Museum of Science and Industry, Portland.
Available at: http://tinyurl.com/n9u3cq3

Guberman, S. R., Bernstein, D., King, Z., Onkka, A., Ostgaard, G., & Van Cleave, S. (2011, March). *Math Moves! Formative evaluation report, Spring 2011.* Unpublished manuscript, Science Museum of Minnesota, Saint Paul.

Gyllenhaal, E. D. (2006). Memories of math: Visitors' experiences in an exhibition about calculus. *Curator: The Museum Journal, 49*(3), 345-364.

Gyllenhaal, E. D., Mackinney, L. H., & McClafferty, T. (1999). *Revisiting Perry's knowledge hierarchy model.* Paper presented at the Visitor Studies Conference, Chicago.

Hall, R., & Nemirovsky, R., (2012). Introduction to the special issue: Modalities of body engagement in mathematical activity and learning. *Journal of the Learning Sciences*. *21*(2):207-215.

Leinhardt, G., Crowley, K., & Knutson, K. (Eds.). (2002). *Learning conversations in museums*. Mahwah, NJ: Lawrence Erlbaum Associates.

Lindgren-Streicher, A. (2011). Findings from universal design professional review, March 24, 2011. Unpublished manuscript, Museum of Science, Boston.

Mackinney, L. H., & Bjork, L. (1999). Quantifying visitors' understanding of earthquakes. *Current Trends in Audience Research and Evaluation, 12*, 42-52.

McClafferty, T. P. (1996). *Empowering your visitors: Improving the visitor's experience through evaluation.* Paper presented at the Museums Australia Conference, Australia.

McClafferty, T. P., & Rennie, L. J. (1997, March). *A triangulation strategy to measure children's learning outcomes from an interactive exhibit.* Paper presented at the National Association for Research in Science Teaching, Chicago.

McManus, P. M. (1987). It's the company you keep...:The social determination of learning-related behaviour in a science museum. *The International Journal of Museum Management and Curatorship, 6*, 263-270.

McManus, P. M. (1988). Good companions: More on the social determination of learning-related behaviour in a science museum. *The International Journal of Museum Management and Curatorship, 7*, 37-44.

McManus, P. M. (1990). Watch your language! People do read labels. In B. Serrell (Ed.), *What research says about learning in science museums* (Vol. 1, pp. 4-6). Washington, DC: Association of Science-Technology Centers.

Miller, C., & Swift, K. (1980). *The Handbook of Nonsexist Writing*. New York: Harper and Row.

Nemirovsky, R., & Ferrara, F. (2009) Mathematical imagination and embodied cognition. *Educational Studies in Mathematics*. *70*(2): 159-174

Nemirovsky, R., & Gyllenhaal, E. D. (2006, January/February). Handling calculus: Graphing motion to understand math. *ASTC Dimensions*, 13-14.

Nemirovsky, R., Kelton, M. L., & Rhodehamel, B. (2013). Playing mathematical instruments: Emerging perceptuomotor integration with an interactive mathematics exhibit. *Journal for Research in Mathematics Education*. *44*(2): 372-415.

Nemirovsky, R., Kelton, M. L., & Rhodehamel, B. (2012) Gesture and imagination: On the constitution and uses of phantasms. *Gesture, 12*(2), 130-165

Perry, D. L. (1993). Measuring learning with the knowledge hierarchy. *Visitor studies: Theory, research and practice: Collected papers from the 1993 Visitor Studies Conference*, *6*, 73-77.

Perry, D. L. (2002). Profound learning: Stories from museums. *Educational Technology, 42*(2), 21-25.

Perry, D. L. (2012). *What makes learning fun? Principles for the design of intrinsically motivating museum exhibits*. Lanham, MD: AltaMira Press.

Rau, J. W. (1990). The professional development of instructional developers through a co-development group process. Unpublished doctoral dissertation, Indiana University, Bloomington.

Rennie, L. J., & McClafferty, T. P. (1996). *Handbook for formative evaluation of interactive exhibits*. manuscript. Curtin University of Technology. Perth, Western Australia.

Selinda Research Associates. (2013). *Math Moves! project summary at the end of year two*. Unpublished manuscript, Chicago.

Science Museum of Minnesota. (n.d.). *Math Moves! Experiencing ratio and proportion: Educator Guide*. Unpublished manuscript, St. Paul. Available at: www.mathmoves.org

Spock, M., Perry, D. L., & Lewis, S. (1997). *Philadelphia stories; Listening to how museum professionals describe pivotal learning experiences in museums.* Unpublished manuscript, Chapin Hall Center for Children at the University of Chicago, Illinois.

Tatter, P. (2004). As Happy as Can Be: Fostering Inclusiveness at Explora
Available at: http://astc.org/pubs/dimensions/2004/nov-dec/index.htm

Tatter, P. (2008). A summary of ideas about transactivity.
Available at:
http://tinkering.exploratorium.edu/sites/default/files/sites/default/files/pdfuploads/a_summary_of_ideas_about_transactivity-2.pdf

Wolf, R. L., & Tymitz, B. L. (1981). *"Hey mom, that exhibit's alive": A study of visitor perceptions of the Coral Reef Exhibit* (Unpublished manuscript). Washington, DC: National Museum of Natural History, Smithsonian Institution.

Wright, T., & Parkes, A. (2015, March/April). Exploring connections between physical and mathematical knowledge in science museums. *Informal Learning Review, 131*, 16-21.

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Appendix A: List of *Math Core* Personnel[14]

Center for Research in Mathematics and Science Education at San Diego State Univ. Ricardo Nemirovsky* Molly Kelton* Bohdan Rohdamel **Explora** Paul Tatter* Betsy Adamson* Armelle Casau* Cobie Howard George Moran Andrea Deets Jeff Taylor **Museum of Science** Alana Parkes* Anna Lindgren-Streicher* Peter Ford* **Museum of Life + Science** Troy Livingston* Elizabeth Fleming* Roy Griffiths*	**Science Museum of Minnesota** J. Newlin* Keith Braafladt* Kirsten Ellenbogen Richard Gagnon Dan Miller* Bette Schmit* Dave Bailey Ben Amel Aaron Heidgerken Maija Sedzielarz Steven Guberman* **Selinda Research Associates** Deborah Perry* Eric Gyllenhaal Diane White **TERC** Tracey Wright* * part of core team (18 members)

Advisors
Hyman Bass, Professor of Mathematics and Mathematics Education, University of Michigan
Karyn Bertschii, Senior Exhibit Developer, Oregon Museum of Science and Industry
Bárbara Brizuela, Associate Professor of Education, Tufts University
Marta Civil, Professor of Mathematics, University of Arizona
Tsivia Cohen, Director of Family Learning Initiatives, Chicago Children's Museum
Paula Hooper, Science Educator/Learning Research Scientist, Exploratorium
Suzanne Perin, candidate in Informal Education Research, University of Washington
Liza Reich Rawson, Project Director, Brooklyn Children's Museum
Chris Robinson, High School Mathematics Teacher, Blake School, Minneapolis
Tom Rockwell, VP of Center for Pubic Exhibition, Exploratorium
Jeanne Vergeront, Museum Consultant, Vergeront Museum Planning
Walter Waranka, Accessibility Advisor

Professional Development Participants
Holly Denman, Educator, Chicago Children's Museum
Rachel Schiff, Research Associate, Randi Korn & Associates, Inc.

[14] All titles and affiliations are during the time of the study. Some may have changed since then.

Appendix B: Exhibit Selection Criteria

These criteria were used to guide the selection of final exhibits to comprise the core set.

10/7/2010
jn

Math Core
Exhibit Selection Criteria

DRAFT

Our goal is to select 6 exhibits to be replicated for installation in four museums: Explora, Museum of Life + Science, Museum of Science, Science Museum of Minnesota.

Each museum may supplement the 6 selected and replicated exhibits with additional exhibits that, in that museum's opinion, support the goals of the project.

The Experience Station will be replicated for installation at each museum.

Filter

Candidate exhibits will meet these criteria.

1. They provide significant experiences with ratio and proportion
2. They are open-ended and accessible.
3. They can be supplied and maintained by all four host museums at least through the calendar year 2014.
4. They pass (or can be modified to pass) a safety review at SMM.

Selection Criteria

1. The exhibit meets as many as possible of these criteria:
    a. it facilitates kinesthetic learning
    b. it provokes and supports math conversations
    c. it provides physical evidence of prior use
    d. it invites and supports parental engagement
2. Evaluation shows that the exhibit attracts visitors and successfully engages them for a significant period of time in math exploration.
3. The exhibit is recommended by at least half of the Math Core Advisors
4. The exhibit fits with other selections to support a diversity of math content and approach.
5. The exhibit fits with other selections into the Math Core replication budget.
6. The exhibit is, in the opinions of the six principal partners, all around excellent.

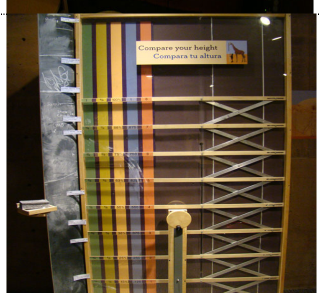
Appendix C: List of All the *Math Moves!* Exhibits

Following is a list of all of the exhibit units that respondents visited during their study visits. Note that these are not all of the exhibits that were developed as part of the *Math Core* project, only those on the floor during the study visits. Note also that some of the exhibits included below were not officially part of a *Math Moves!* exhibition. They are listed here because RRs engaged with them during their visits and considered them part of *Math Moves!*.

#	Respondents' Nicknames	Image	EXP	MLS	MoS	SMM	MCM Title of Exhibit	Notes
1	Rainbow		X	X	X	X	Partner Motion	Core Exhibit
2	Sliders		X	X	X	X	Sensing Ratios	Core Exhibit
3	Shadow Table		X	X	X	X	Shadow Fractions	Core Exhibit
4	Blocks		X	X	X	X	Scaling Shapes	Core Exhibit

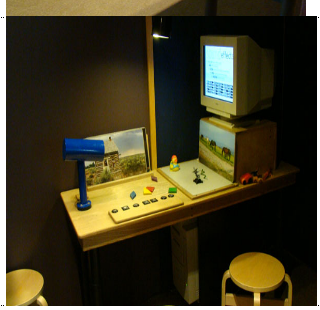
#	Respondents' Nicknames	Image	EXP	MLS	MoS	SMM	MCM Title of Exhibit	Notes
5	Clicker		X	X	X	X	Comparing Frequencies	Core Exhibit
6	Balance		X	X	X	X	Balance & Imbalance	Core Exhibit
7	Chairs			X	X	X	Comparing Forms	Core Exhibit
8	Spirograph		X			X	Drawing With Gears	Core Exhibit
9	Comment Board		X	X	X	X	Feedback Station	Core Exhibit

#	Respondents' Nicknames	Image	EXP	MLS	MoS	SMM	MCM Title of Exhibit	Notes
10	Shapes from Circles					X	Shapes from Circles	MCM Exhibit
11	Big Shadows			X		X	Half Whole Double	MCM Exhibit
12	Slopes					X	Slopes Math Moment	MCM Exhibit (prototype)
13	Triangles					X	Triangle Math Moment	MCM Exhibit
14	Lever		X				Feel the Force	MCM Exhibit

#	Respondents' Nicknames	Image	EXP	MLS	MoS	SMM	MCM Title of Exhibit	Notes
15	Mirrors		X				Sliding Mirrors	MCM Exhibit
16	Pulleys		X				Speed Comparator	MCM Exhibit
17	Moving Shadows		X				Moving Shadows	MCM Exhibit
18	Heights		X				Comparing Your Height	MCM Exhibit
19	Drawing		X				Drawing Proportional Shapes	MCM Exhibit

#	Respondents' Nicknames	Image	EXP	MLS	MoS	SMM	MCM Title of Exhibit	Notes
20	Pattern Generator		X				Combining 2 Straight Movements to Create Curves	MCM Exhibit

Non-*Math Core* Exhibits

#	Respondents' Nicknames	Image	EXP	MLS	MoS	SMM	Title of Exhibit	Notes
21	Color Spinner		X				Color Ratios	In-house exhibit located within/adjacent to *Math Moves!*
22	Magni-Cam		X				Magni-Cam	In-house exhibit located within/adjacent to *Math Moves!*
23	Animation		X				Animation	In-house exhibit located within/adjacent to *Math Moves!*

#	Respondents' Nicknames	Image	EXP	MLS	MoS	SMM	MCM Title of Exhibit	Notes
24	Magnets		**X**				Strong Magnets	In-house exhibit located within/adjacent to *Math Moves!*

Appendix D: Evaluation Plan

The following pages contain the complete evaluation plan that served as the blueprint for the summative evaluation study

Math Moves!
Summative Evaluation Plan

2/10/12

Description of What's to be Evaluated
This study will evaluate the effectiveness of four *Math Moves!* exhibitions at engaging visitors with concepts related to ratio and proportion in rich and meaningful ways over time. Each exhibition will have between 9 and 15 exhibit units that were developed as part of the NSF-funded *Math Core* project. These units will be installed at the four partner museums—Science Museum of Minnesota, Museum of Science in Boston, Explora in Albuquerque, and Museum of Life and Science in Durham, North Carolina—in January 2012. The exhibitions are permanent—i.e. not traveling.

Each museum will have an identical core set of 5 exhibit units: Comparing Frequencies, Partner Motion, Scaling Shapes, Shadow Fractions, and Sensing Ratios, in addition to a paper and pencil Feedback Station. The remaining 3-9 exhibit units at each site have been selected by each host museum from the exhibits developed as part of the *Math Core* project. The specific floor plan for and location of each *Math Moves!* exhibition will be determined entirely by the host museum. In addition to the exhibit units, a *Math Moves!* website and educational programming and materials are also being developed. Although these non-exhibit *Math Core* components may be briefly reviewed as part of the study, they will not be a major focus.

Some museums may opt to develop and include additional (non-*Math Core*) exhibit units, and some institutions may develop (again non-*Math Core*) accompanying programming, special events, and/or online or social media presences. These additional supplementary aspects may be as part of the study as appropriate, although they will not be a major focus.

The focus of this study will be visitor experiences at the four *exhibitions* (as they are defined by each institution). The study will not evaluate the individual exhibit units, although it will likely include discussions of visitor experiences at specific units.

Type of Evaluation and Research Question
This evaluation study will be a longitudinal summative evaluation study to examine the effectiveness of the *Math Moves!* exhibitions at achieving the goal of engaging museum visitors with math concepts related to ratio and proportion over time.

The primary research question is:

> In what ways and to what extent do the four *Math Moves!* exhibitions contribute to the development of repeat visitors' understanding of, appreciation for, and fluency* with ratio and proportion (and related math concepts) as they recurrently engage with the exhibitions over two years?

A secondary question relates to the contribution of *Math Core* to professional development:

> In what ways and to what extent did participating in this project contribute to the evolution of museum professionals' thinking about (a) the development of math-related STEM visitor experiences; (b) the development of ISE opportunities for repeat visitors; (c) their own understandings of and relationships with concepts related to math, ratio, and proportion; and (d) the power and efficacy of collaborative projects.

* It's important to note that in this study we are interested in a very broad definition of "understanding, appreciation, and fluency." We are using these terms to encompass many types of learning over time, including verbal, academic, affective, attitudinal, emotional, visceral, kinesthetic, behavioral, social, and experiential. These three overarching categories are not—and are not intended to be—mutually exclusive but rather attempt to define a very broad world of learning, regardless how it manifests itself. Note also that *fluency* in this case does not refer to complete mastery, but rather a sense of being comfortable and at ease with operations. Specific indicators of the various types of learning identified above will be described in detail in the topical framework.

The study will focus on the ability of the exhibitions to provide on-going, rich and meaningful experiences that evolve over time, not on the ability of the exhibitions to attract repeat visitors. In other words, we won't be focusing on "Are the exhibits effective at attracting repeat visitors?" but rather "When people visit over time, how are the exhibits effective at helping them evolve their understandings of ratio and proportion?"

Finally, while not a primary focus of the study, we will not exclude first time or other visitor experiences that do not take place over time. We believe that looking at these briefer and more isolated experiences will provide additional insights into the visitor experience over time, as well as contribute to our understanding of the overall effectiveness of the exhibitions.

Roles & Responsibilities

<u>Core Evaluation Team (CET)</u>

The core summative evaluation team will consist of four individuals: J. Newlin, Deborah Perry, Tracey Wright, and Steven Guberman. All members of the CET will participate in all meetings, review all documents, and give timely feedback. They will all contribute to the collaborative decision-making process.

J. will serve as client for the summative evaluation project. As such, he will be the primary representative of the larger *Math Core* project and will serve as the liaison between the *Math Core* project and the evaluation study. J. has the responsibility and authority to make decisions on behalf of the Science Museum of Minnesota. He will receive the final research report(s).

Deborah will serve as project manager and lead researcher. As project manager, she will make sure the study progresses as planned, is of high quality, and stays within budget and on time. As lead researcher, she will be responsible for all data collection, analysis, and report writing, although additional researchers may be brought in to participate in many of these tasks.

Tracey Wright and Steven Guberman will serve as research and evaluation advisors. In addition to participating in all meetings and joint decision-making, they will review documents, provide feedback, and help keep the big picture in mind.

Museum Rep Team (MRT)
In addition to the Core Evaluation Team, there will be an advisory group comprised of the following individuals:
- Anna Lindgren-Streicher (Museum of Science, Boston)
- Betsy Adamson (Explora, Albuquerque)
- Elizabeth Fleming (Museum of Life + Science, Durham)
- (Steve Guberman & J. Newlin will fulfill this role for the Science Museum of Minnesota)
- Ricardo Nemirovsky (San Diego State University)

The roles and responsibilities of members of the MRT are as follows:
1. Help get evaluation buy-in and a sense of ownership from their particular institution.
2. Facilitate the evaluation study by (for example) recruiting respondents, training floor staff in how to recruit respondents, acting as a liaison to the recruited families and the researchers, and serving as the primary contact/logistics person when Selinda makes site visits to collect data.
3. Facilitate communication about the evaluation study by (for example) distributing draft evaluation documents for feedback, compiling feedback, and ensuring *Math Core* development staff are aware of evaluation opportunities and findings.
4. Additional roles and responsibilities will emerge through the course of the project.

Math Core Project Team (MPT)
This team is comprised of the museum partners, advisors, and other players who have participated in the *Math Core* project. It includes all of the members of the Core Evaluation and Museum Rep Teams, as well as: Alana Parkes, Betsy Schmit, Dan Miller, Keith Braafladt, Kelly Marks, Kirsten Ellenbogen, Maija Sedzielarz, Molly Kelton, Paul Tatter, Roy Griffiths, and Troy Livingston. As members of the MPT, these individuals will receive and review draft evaluation documents and give feedback.

Communications & Relationships
The Core Evaluation Team (CET) is committed to a collaborative process where expertise, information, and concerns are shared, and decisions are jointly made. Depending on the activity, the major responsibility for a particular task might rest with the client, project manager/lead researcher, or research/evaluation advisors.

To maintain frequent and open communication, we will have periodic phone meetings throughout the duration of the study. Most meetings will be held on either (or both) the first and third Thursday of the month at 3pm/4pm (central/eastern). All members of the CET will participate in all meetings. Special meetings with the Museum Rep Team (MRT) will be called as necessary. All members of the *Math Core* Project Team (MPT) are invited to participate in any or all of the evaluation meetings.

Methodology
This study will use a naturalistic inquiry methodology (Lincoln & Guba, 1985; Allen et al., 2007). Naturalistic methodology studies real-life, naturally occurring environments and phenomena, and tends to rely on qualitative data such as observations and interviews. It uses mixed methods to examine the situation being studied from as many different perspectives as possible. Naturalistic inquiry tends to rely on an emergent research design, and is primarily descriptive rather than prescriptive.

Developmental Evaluation
It should be noted that while working on this evaluation plan we explored the feasibility of conducting a developmental evaluation instead of a summative evaluation (Patton, 2011). Although appealing in many ways—especially because of its emphasis on examining complex environments, reliance on flexible and adaptive research designs, and frequent reporting of results—the primary focus of developmental evaluation is the development of a new product or model. Because resources for continued development of *Math Core* are currently non-existent, we decided instead to move forward with a summative evaluation, but to incorporate aspects of developmental evaluation, as they seem appropriate. If at some point supplemental funding becomes available to engage in ongoing development, we may revisit this decision.

This summative evaluation study is in alignment with developmental evaluation in these ways:

- Research design is flexible, evolving, and adaptive over time; not pre-determined.
- Reporting of results and findings takes place rapidly and frequently throughout the study.
- Allows for documentation of some of the messy stuff including: the implementations of the *Math Moves!* exhibitions, as well as the evolution of ideas and thinking about (a) museums doing math exhibits and (b) how to support repeat visitors.
- What is being studied is complex and changing.

This study is not in alignment with developmental evaluation in these ways:

- There is no product being developed, nor any expectation of continued evolution or development of exhibits or programming.
- Partners' obligations to the exhibitions are pretty much completed.
- Developmental evaluation presupposes constant iteration. We don't have funding currently to engage in extensive iteration and development, but are keeping tabs on the possibility of supplemental funding opportunities.

Study Design
An important component of this summative evaluation will be an emergent research design. This is in accordance with standards for conducting naturalistic inquiry and, we believe, will give us the flexibility necessary to respond to data as it is being gathered and to findings as they emerge. For example, rather than pre-determining the specific number of respondents, interviews, or even issues to be explored, these will be dictated by the needs of the project and progress of the data collection. We may find for example, that one respondent group is contributing particularly useful perspectives so we may want to spend more time with them. Or we may find that we are getting more useful information through phone interviews so we may conduct fewer site visits. Throughout the two years spent gathering data, a modified inductive

constant comparison approach (Lincoln & Guba, 1985) will be used to analyze the data and re-assess what is being found out and what issues need to continue to be explored.

While we acknowledge that the final design will emerge during the study, it is currently anticipated that the summative evaluation will likely include four major components briefly outlined below. As things progress, the design will continue to evolve and be refined based on what emerges from the data.

1. **Site Visits**: Conduct site visits to each partner institution to see their *Math Moves!* exhibition, and observe and interview visitors using the exhibition. We anticipate that two 3-day site visits will be made to each partner institution. These site visits will take place shortly after the installation during the spring 2012, and again towards the end of data collection during the summer and/or fall 2013. The focus of the site visits will be to (a) see the installations, (b) observe and interview casual visitors using the exhibit units, and (c) observe and interview specially recruited repeat visitors (see <u>Longitudinal Observations and Interviews</u> below). It should be noted that ideally more than two site visits to each institution would be made. In the end, we will need to determine the most cost effective way to get the most data. We'll have a better sense of this after the first round of site visits that will take place in the spring 2012.

2. **Longitudinal Observations and Interviews**: Follow (via observations, phone/video interviews, written surveys, journals, and drawings) the experiences of 2-5 family[1] groups at each institution as they engage with the exhibitions over a 2-year period of time. Because this study is interested in how the *Math Moves!* exhibits help visitors evolve their understanding of and fluency with math concepts over time, part of the study will include following specially recruited repeat visitors.

3. **Staff Interviews**: Interview (via phone) 8-10 museum staff or other professionals who participated in the *Math Core* project. Most of these interviews will take place at the beginning of data collection while participating in the collaborative process is still fresh in their minds. It is anticipated that a few of these interviews will be conducted towards the end of data collection.

4. **Review of Documents**: Review selected written documents related to the *Math Moves!* exhibitions, accompanying materials, and online presences. SRA researchers will regularly monitor and review documents throughout the two years of data collection. This is described in more detail below under *Data Collection Methods*.

Topical Framework

During the initial planning phases of the study a detailed topical framework will be developed. The topical framework is a list of topics, or issues, that we will explore as part of this study. It is *not* the questions we will ask of respondents, but rather the questions we want to find the answers to in order to answer the larger research questions identified above. The topical framework will be collaboratively developed by the Core Evaluation Team with input from the Museum Rep Team,

[1] While respondents will sometimes be referred to as family groups, *family* will be loosely defined to include any group that remains intact over the two-year period.

and the *Math Core* project team members. While the topical framework is still being developed, it is anticipated that it will include a number of issues including the four types of overlapping (i.e. not mutually exclusive) engagements: physical, emotional, social, and intellectual.

Physical engagements include all the many physical ways that visitors engage with the exhibitions. They include various ways that visitors behave including: where and how much time is spent with different exhibits; what they touch and manipulate; where they look, stand, sit, and lean; if and what they read; what order they do things in; physical gestures that embody math; etc.

Social engagements include all the many ways that visitors engage with one another and with museum staff to make meaning while they are in the exhibitions. They include activities such as visitors asking and answering questions; directing someone's attention either verbally or through gestures; explaining something to someone else; putting their heads together to figure something out; using gestures and body movement to help explain something or ask a question; wondering out loud; verbalizations such as "Look at the square over there. Notice how it is getting larger. Let's try to make it get smaller," etc.

Emotional engagements include all the many emotional ways visitors engage with the exhibitions including: finding enjoyment; experiencing frustration; being amazed or excited about something; being comfortable and at ease; using gestures and body movement to express any of these emotional experiences; verbalizations such as "This is so cool," or "This is boring," etc.

Intellectual engagements include all the many cognitive and intellectual ways visitors engage with the exhibitions. Intellectual engagements include: having an *aha! experience*; pondering something; expressing awe, surprise, or intrigue; using gesture and body movement such as smacking one's forehead, shrugging one's shoulders, or physically acting out a problem or explaining a math concept; wondering out loud; verbalizations such as "I got it" or "I don't get it," etc.

Data Collection Methods
While the specific data collection methods will be determined by the needs of the research as it progresses, it is anticipated that they will include at least unobtrusive observation, depth interview, and document review, but may include additional strategies (Lincoln & Guba 1985, p. 267-281). In accordance with standards for naturalistic inquiry, it should be noted that most of the data collection methods will be open-ended and will focus on holistic understandings of the issues being explored. In other words, in looking at (for example) physical engagements, researchers will focus on the gestalt of the physical interactions rather than identifying, counting, or measuring specific behaviors.

Unobtrusive observation: During an unobtrusive observation, researchers stand back and watch visitors as they engage with an exhibit or exhibition, taking notes and often using an observation protocol that outlines specific things to look for, especially those items identified in the topical framework. While they are conducting the observation, they try to remain unnoticed by the respondent group, but don't go out of their way to disguise or hide themselves. At the end of the observation, the researcher finds a quiet location to *debrief* (described below under *Data*

Analysis Methods). While educational programming will not be a major focus of this study, if programming is taking place during site visits, these programs may be observed.

Extended observation: Similar to unobtrusive observations, extended observations have the researcher at a distance from and generally unnoticed by visitors using an exhibit unit. Instead of focusing on an individual respondent group, in an extended observation the researcher will observe all the visitors to one selected unit for an extended period of time, for example 30 minutes. Extended observations give a quick snapshot of how visitors tend to use a particular unit, e.g. how long they stay and what they do.

Depth interview: A depth interview is a specific type of open-ended interview usually conducted face-to-face but also sometimes conducted over the phone, and possibly using video technology such as Skype. It is unstructured and conversational in nature. Although it may start with a list of general questions or issues to explore, it tends to follow the lead of the respondent and what the respondent feels is important. In this way, the depth interview is able to elicit findings according to the respondent's framework, perspective, or worldview. Depth interviews usually begin by establishing rapport with the respondent, and they tend to be lengthy in nature. It is not unusual for a depth interview with a casual museum visitor to last 15-30 minutes or longer, and phone interviews with museum professionals often last an hour or more. Most face-to-face depth interviews with casual museum visitors will take place on the floor of the museum immediately following an unobtrusive observation. As with observations, at the end of the depth interview, the researcher finds a quiet place to write a debrief.

Participant observation: Participant observations are unlike unobtrusive observations in that the researcher becomes part of the respondent group. Often this happens by engaging with a visitor group naturally as they use an exhibit unit. After sufficient rapport building, the researcher asks permission to "join" the group. While now a member of the group, the researcher maintains a certain distance, following the lead of others, but asking questions, primarily clarification questions, as appropriate. During participant observations the respondent often joins in the preliminary data analysis by helping clarify researcher observations. At the end of a participant observation, the researcher writes a debrief before collecting additional data.

Journaling: This technique has respondents write regular memos or journal entries over a particular period of time. Ideally journaling includes not just written notes but also photographs, drawings, artifacts (such as a flyer, button, or souvenir), video clips, blog postings, or other creative endeavors. In this study we may use journaling with selected museum staff and/or recruited respondents. For recruited respondents, it may include a form for respondents to complete at the end of each visit to the museum.

Document review: Documents and records can sometimes provide additional important information. In this case "the term 'document' is used to denote any written or recorded material…that was not prepared specifically in response to a request from the inquirer (such as a test or set of interview notes)" (Lincoln & Guba, 1985, p. 277). Documents may include flyers or other promotional materials, the project website, other related websites, program materials, statements of goals and objectives, etc.

Data Analysis Methods
Constant comparison: Data analysis will use a modified inductive constant comparison approach (Lincoln & Guba, 1985, p. 339-344) whereby each unit of data is systematically compared with each previous and subsequent unit of data. The data analysis is inductive in that the categories and findings emerge from the data, rather than being pre-determined ahead of time. While the topical framework pre-determines many of the issues we are interested in exploring at the beginning of the study, by employing inductive data analysis we create the space for additional findings to emerge. Because this is an evaluation study, and because our time on site will necessarily be limited, we will strive for efficiency and flexibility during data analysis, especially during site visits.

The Debrief: In naturalistic inquiry, data analysis is ongoing throughout the data collection process and begins with debriefing. After each data collection event (e.g. an observation or interview), the researcher finds a quiet location to conduct a debrief. During this initial debriefing process, the researcher who conducted the interview or observation fleshes out and clarifies any notes taken and then reflects on what was observed or discussed, comparing and contrasting these data to all previous data in order to identify trends and contradictions, to develop and clarify preliminary understandings and evolving findings, and to help determine the focus of the next observation. The written debrief includes a brief summary of the event, a summary statement of what was found out, a statement of where these findings contradicted or raised questions about previous data, and a list of new questions or issues that were raised. Periodically throughout the study, additional group debriefing sessions are held, usually involving other researchers who have collected data and, in some cases, additional stakeholders (such as the members of the three teams identified in this document).

Description and Selection of Respondents
Respondents for this study will be purposively (rather than randomly) selected, and in accordance with standards for conducting naturalistic inquiry, the specific number of respondents won't be pre-determined. In naturalistic inquiry, data are usually collected until redundancy is reached. Redundancy is when no new findings emerge in spite of repeated efforts to elicit new information. In this study, rather than trying for a particular sample size, we will strive for (a) maximum diversity, including Spanish-speaking participants, and (b) prolonged engagement, i.e. contact hours with respondents.

In defining diversity, we are less concerned with making sure we have individuals from a set of pre-determined categories and more interested in, at the end of the study, being able to look back on the respondents and say that neither of the two groups (visitor and professional) is homogeneous or representative, but rather encapsulates as wide a range of experiences as possible. Some examples of diversity might include the following: respondents covering a range of (a) museum-going and/or exhibit development experience, (b) experience with and attitudes towards math, (c) racial/ethnic backgrounds, (d) socio-economic status, (e) ages, and (f) people with disabilities. Because the exhibits are in both English and Spanish, we will strive to include some families who are fluent in Spanish.

There will be three types of respondents: (1) casual visitors, (2) recruited visitors, and (3) museum professionals.

Casual Respondents (CR) are those visitors who are have come to the museum without knowing about the evaluation study. They may or may not have known ahead of time about the exhibition, but they are in the exhibition because they decided (on their own) to visit it. Ideally most of these respondents will be repeat visitors, but we will also include many who are first-timers. These respondents will be observed when possible, and may be invited to participate in a depth interview to share their experiences.

It is anticipated that most CRs will be identified and selected as part of regular purposive sampling by Selinda researchers during the site visits. However, it is likely that some CRs will be identified by museum staff. For example, a member of the floor staff at a museum might come to recognize a particular repeat visitor over the months (or years) the exhibition is on the floor. Or they might overhear a visitor conversation, for example someone saying to their visiting companion that "something is different" from all the times they've been there before, or they "learned something knew this time 'round." These visitors will be invited to fill out a brief form with their email address, which will then be submitted to the Museum Rep at that institution. That person will follow up with them by contacting them and then conducting a phone or email interview. Depending on their experiences and interest, at that time they might also be invited to participate as a Recruited Respondent (see below). Casual Respondents will be selected from among all visitors to the exhibitions, and, will not be limited to those of the target age, nor repeat visitors.

Recruited Respondents (RR) are those museum visitors who are invited to participate across the multi-year evaluation study. Early in the study, each institution will recruit 5 potential respondent groups to be considered for participation in the study. Using purposive sampling, a total of 2-5 of these groups at each institution will be purposively selected by the CET, for a total of 8-20 recruited respondent groups. Additional RRs may be added later during the study.

Each RR group will make at least 6 visits (3 during each year of data collection) to the museum. Each visit will be followed up by a written activity (e.g. survey, journal, drawing) and/or a depth interview. When possible, the interviews will be conducted by an SRA researcher during site visits. When this is not possible, they will be conducted by phone and/or video conference such as Skype.

An adult member of each Recruited Respondent group will sign a contract indicating certain criteria and requirements for participating in the study. While this list is still being developed, it is anticipated that it will include items such as the following: they (1) have access to a computer and are comfortable using email; (2) have at least one child 6-11 years old; (3) can commit to visit the museum at least three times during each of two years – a total of six visits; (4) can commit to completing a (30-minute) written survey/journal/drawing exercise after most visits; (5) can commit to participating in a face-to-face or telephone interview (up to one hour) after most visits. A "recruitment packet" including consent forms and a contract will be developed to explain requirements in detail.

While RR groups can be as large as the family wishes, one individual (a child between the ages of 6-11) will be identified as the *focus child*, i.e. the primary person we will be following, and one will be identified as the *accompanying adult*. Each study visit must include at least the focus child and the accompanying adult. We recognize however that these respondents may be part of a larger social group and, because this is a naturalistic study, we will gather data within the context of the entire visiting group, however it is configured during each visit.

The four host museums will be responsible for recruiting all Recruited Respondents. This might include (a) identifying repeat visitors they already have a relationship with, (b) friends and neighbors, (c) students in school, (d) visitors they observe during the early days of the exhibition that seem to be particularly enjoying the exhibition, (e) holding a special math appreciation night, (f) including announcements about the study in museum publications for members and others, and (g) working with local community agencies.

Museum Professional Respondents (MPR) are those who have been active in the *MathCore* project in some capacity. They will be purposively selected and invited to participate in depth telephone interviews. It is anticipated that there will be approximately 8-10 Museum Professional Respondents.

Compensation
All respondents will have the knowledge that they are contributing to an important piece of research. Casual Respondents will receive a small token of appreciation, usually an item from the gift shop ($2-$5) or a gift certificate. Approximately 30 tokens will be supplied by each museum for use by the researchers during each site visit.

Recruited Respondents will receive additional items to facilitate their fulfilling the terms of the contract. While this list is still being developed, it is anticipated that compensation may include the following:

- unlimited family membership to the museum during the two year period of data collection
- free parking or assistance for transportation costs such as bus fare, or mileage
- voucher for a meal in the cafeteria
- copy of the final report(s) if interested

Products
The typical product or deliverable for a study such as this is a final evaluation report. With this study we have agreed that more frequent reporting will be desirable. While the details are still being worked out, we anticipate that the products for this study will include the following:

- Three annual status reports submitted in May 2012, 2013, 2014. These status reports will briefly describe the activity to date and outline the next steps.
- A series of *evaluation memos* that periodically present a summary of findings to date and invite feedback and comments from the larger group. These evaluation memos will be narrative (as opposed to statistical) and will include findings in visitors' own words. It is anticipated that evaluation memos will be written after each round of site visits.
- A final culminating stand-alone mini-report or article for publication. This final article will include a brief overview of methods, findings, and a section on Lessons Learned. While useful for NSF, its primary audience will be people in the museum field.

Timeline

June – December, 2011	Planning for the study
December, 2011	Exhibits shipped from SMM to three museum venues
January – February 2012	Exhibits installed at four museums
January – December, 2012	Data collection and analysis
January 2013	Working Project Meeting at Museum of Science
January – December, 2013	Data collection & analysis
January – July 2014	Write final mini-report/publication
August 15, 2014	Final mini-report/publication due to SMM

Operating Budget

Approximately 108 people days and $18,600 in travel, transcription, supplies

Logistics

Before each site visit, the researcher will contact the Museum Rep, and they will work together to discuss (a) the best date(s) for the site visit; (b) options for lodging; (c) access to the museum; (d) scheduling meetings with Recruited Respondents; (e) securing tokens of appreciation for Casual Respondents; (f) posting of signs about data collection; (g) special events or circumstances that will be taking place during the site visit; (h) any additional information that might be appropriate.

Ethical Treatment of Respondents

Selinda Research Associates and all four participating museums are committed to the ethical treatment of respondents. Respondents' confidentiality will be strictly maintained, and we will work hard to minimize any disruption of visitors' visits or professionals' work. The design for this study will be submitted for an IRB review by the Museum of Science in Boston. We will work with Anna Lindgren-Streicher to ensure all forms are completed and requirements fulfilled.

Dissemination of Reports

All reports, publications, and evaluation briefs will be shared as broadly as possible, and posted online when appropriate. Reports and briefs will be posted on (a) the project's Ning site: http://mathcore.ning.com/, (b) Selinda's website: www.selindareesearch.com, (c) www.informalscience.org, and (d) others yet to be identified.

Project Closure

Selinda Research Associates is committed to reflective practice and learning from our experiences. When this evaluation study is completed, the Core Evaluation Team will have a project closure meeting to reflect on the overall *Math Moves!* summative evaluation project. It should be noted that this is not an evaluation of the evaluation, but rather an opportunity for everyone involved in this study to step back and reflect on what went well, as well as the challenges we encountered so that we may all get smarter about evaluation in informal settings.

References Cited

Allen, S., Gutwill, J., Perry, D. L., Garibay, C., Ellenbogen, K. M., Heimlich, J. E., et al. (2007). Research in museums: Coping with complexity. In J. H. Falk, L. D. Dierking & S. Foutz (Eds.), *In principle, in practice: Museums as learning institutions* (pp. 229-245). Lanham, MD: AltaMira Press.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage Publications.

Patton, M. Q. (2011). *Developmental evaluation: Applying complexity concepts to enhance innovation and use*. New York: The Guilford Press.

Williams, D. D. (n.d.). *Educators as inquirers: Using qualitative inquiry.* Unpublished manuscript, Brigham Young University, Provo, UT.

Appendix E: Topical Framework

The following pages contain the complete topical framework that was collaboratively developed during the course of the study. The topical framework outlines all the issues we were interested in exploring, and it evolved through the initial stages of data collection as it was informed by the findings as they began to emerge. While the team was interested in all these issues, not all questions could be pursued.

THE OVERALL EVALUATION QUESTIONS

The primary evaluation question is:

> **In what ways and to what extent did the four *Math Moves!* exhibitions contribute to the development of repeat visitors' understanding of, appreciation for, and fluency[1] with ratio and proportion (and related math concepts) as they recurrently engaged with the exhibitions over two years?**

The secondary question is:

> In what ways and to what extent did participating in this project contribute to the evolution of museum professionals' thinking about (a) the development of math-related STEM visitor experiences; (b) the development of ISE opportunities for repeat visitors; (c) their own understandings of and relationships with concepts related to math, ratio, and proportion; and (d) the power and efficacy of collaborative projects.

Visitors refers to all visitors to the exhibitions, whether they are first time visitors or repeat visitors, casual respondents or recruited respondents, adults or children. In this study, *visitors* does not include museum professionals (unless they are operating primarily as visitors and are not wearing their museum professional hats).

To answer the evaluation questions listed above, we will explore the issues below. This list of questions lays out important issues we want to keep in mind as we are gathering data, but we will not be answering all questions definitively. As data collection proceeds we may find that some questions become less important and others more important. Some questions may drop off the topical framework, and new ones may be added. We will however pay special attention paid to the categories of *Fluency and Knowledge*, *Intellectual Engagements*, and *Kinesthetic Engagements* (sub-category of Physical Engagements). Although not specifically mentioned in many of the categories, we are interested in how most of these issues evolve over time, across settings, and by different audiences. We are also interested in paying attention to the extent to which the Math Moves *exhibits* contributed to the evolution of visitors' understanding and fluency, as opposed to (for example) this being a natural developmental process.

Finally, note that the following categories are not—and are not meant to be—mutually exclusive. Many of the categories overlap, and most include similar—although slightly different perspectives on—questions.

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[1] Fluency in this case does not refer to complete mastery, but rather a sense of being comfortable and at ease with operations.

Math Moves! Summative Evaluation Topical Framework 1/31/13

A. Description of Exhibitions

I. Look & Feel

1. What was the look & feel of each installation?
2. What were the floor plans for each installation?
3. What exhibits were part of each installation, and what did this look like?

II. Supplementary Components

1. What was the online/digital presence at each of the installations?
2. What educational materials were at each of the installations?
3. What special programs and events related to the exhibitions took place?

III. Comparisons

1. How did the floor plans, look & feel, units on the floor, online presences, educational materials, and programs & events compare and contrast among and between installations?
2. How did the exhibits and materials change over time?

B. The Visitor Experience

VISITS

I. Visits and Repeat Visits

1. Who did visitors bring with them during each visit?
2. What were visitors' motivations for making visits and repeat visits, especially with respect to visits beyond the required minimum of six?
3. What role did compensation, expectations, and being part of the study influence visitors' motivations for making repeat visits?
4. How did visitors' visits to the *Math Moves!* exhibits fit in with their visits to the rest of the museum?
5. What was the frequency and pattern of repeat visits? Were there recognizable patterns of use that emerged over time?
6. Were there particular exhibits that some visitors returned to repeatedly? Did certain components get visited repeatedly or regularly? Were there certain exhibits that were visited less frequently or regularly than others?
7. What was the nature of repeat visits? What did these repeat visits look and feel like that was the same or different from other museum visits?
8. What was the role/contribution of repeat visits on visitors' enjoyment, understandings, attitudes, and actions?
9. What was the point at which visitors stopped repeat-visiting? When did they feel "done?"
10. What happened between visits to the exhibition? In what ways and to what extent did visitors think and/or talk about the exhibits between visits?

OUTCOMES

II. Knowledge & Fluency

1. To what extent and in what ways did visitors' understandings of ratio and proportion evolve over time?
2. What was the contribution of the *Math Moves!* exhibition to visitors' understandings of ratio and proportion, and other math-related constructs?
3. What was the relationship between visitors' understanding of math in the exhibitions, math at school, and math in their everyday lives?
4. In what ways and to what extent did visitors' fluency with math, and ratio and proportion evolve?
5. In what ways and to what extent did visitors make connections between embodied understanding of mathematics and more abstract knowledge?
6. In what ways and to what extent did visitors perceive the exhibits to be about math?
7. What language did visitors use to describe what the exhibits were about? In what ways and to what extent was this math-language?

III. Attitudes

1. What were visitors' attitudes towards math? Towards algebra? Towards ratio and proportion? How did these attitudes evolve over time?
2. In what ways and to what extent did visitors become more and less comfortable with math over time?
3. In what ways and to what extent did the exhibits help visitors broaden their understanding of what math is or could be?

IV. Identity

1. What were visitors' long-term memories of math?
2. What contribution did the exhibits make towards visitors' sense of self? How did this evolve over time?
3. What was the nature of visitors' relationships and personal connections with math, and with ratio and proportion?
4. How did visitors' relationships with math—and with ratio and proportion—evolve over time?
5. How did the *Math Moves!* exhibits contribute to visitors' personal ways of learning about and understanding math? What is the landscape of things that contribute to visitors' relationship with and understanding of math?

V. Skills

1. What operational skills with math, algebra, and ratio and proportion did visitors bring to their visits?
2. How did these skills evolve over time?
3. In what ways and to what extent did visitors' skills with mathematical operations in general—and ratio and proportion operations in particular—evolve over time?
4. In what ways and to what extent did these skills overlap with and/or contribute to math classes in school?

Math Moves! **Summative Evaluation Topical Framework** 1/31/13

ENGAGEMENTS

VI. Physical Engagements

1. What exhibit units did respondents use and in what order?
2. How long did they stay at different exhibit units?
3. How did time-at-exhibit compare and contrast by exhibit? By installation?
4. How did visitors engage kinesthetically with the exhibits?
    a. In what ways and to what extent were kinesthetic engagements learning?
    b. What did that kinesthetic learning look like? When was it whole-body, and when was it part-body? When was it something else?
    c. In what ways and to what extent did kinesthetic engagements contribute to the evolution of visitors' understanding & fluency, attitudes, skills, and identity?
    d. How did these kinesthetic engagements evolve over time?
5. When and how did visitors decide to terminate their engagements at each exhibit they used?
6. What were the motivations for leaving an exhibit?

VII. Intellectual Engagements

1. In what ways and to what extent were visitors engaged intellectually?
2. How did these intellectual engagements evolve over time?
3. What role did visitors' memories of math play in their intellectual engagements?
4. What questions did visitors come up with, and what challenges did they generate for themselves?
5. How did these questions and challenges evolve over time?
6. What other ways of intellectually engaging with the exhibits changed over time?
7. How did visitors think about *Math Moves!* between visits? How did they connect their *Math Moves!* experiences to what they learned elsewhere?
8. What language did visitors use to describe their engagements with the exhibits? In what ways and to what extent was this math-language?

VIII. Social Engagements

1. In what ways and to what extent did visitors engage in social engagements while using the *Math Moves!* exhibits?
2. To what extent did visitors engage with the exhibits in social vs. solo ways?
3. What was the nature of these social interactions? In what ways and to what extent were the social engagements part of meaningful teaching/learning experiences?
4. What was the nature of joint meaning-making at the *Math Moves!* exhibits?
5. When did social interactions contribute to the meaningful construction of knowledge and fluency with math, ratio, and proportion?
6. To what extent did this social teaching/learning happen among members of the visiting group? When did it happen with other visitors not part of the visiting group? And when did it happen with floor or interpretive staff?
7. In what ways and to what extent did floor/interpretive staff contribute to/detract from visitor understanding, fluency, attitudes, skills, identity?
8. Did repeat visits tend to be more or less social that initial visits?
9. How did teaching/learning and joint meaning-making evolve over time?
9. What other ways of socially engaging with the exhibits changed over time?

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10. What was the nature of math conversations?
11. In what ways did visitors become more comfortable with math conversations over time?

IX. Emotional Engagements

1. What was the range of ways visitors engaged emotionally with the exhibits?
2. In what ways and to what extent did visitors enjoy their time with the exhibits?
3. Which exhibit units were the most popular? In what ways and to what extent?
4. What did visitors find the most and least enjoyable about their experiences with the *Math Moves!* exhibits?
5. How did visitors' emotional engagements and memories evolve over time?

MOTIVATIONS

X. Motivational Aspects[2]

1. Communication: In what ways and to what extent were visitors able to make sense of the exhibits, manipulables, math concepts, and their experiences with the exhibits?
2. Curiosity: In what ways and to what extent did the exhibits surprise and intrigue visitors?
3. Confidence: In what ways and to what extent did visitors feel safe and smart?
4. Challenge: In what ways and to what extent did visitors feel appropriately challenged?
   a. In what ways and to what extent did visitors expect these exhibits to be about math? About algebra?
   b. What role did visitors' perceptions have on various outcomes such as knowledge & fluency, attitudes towards math, operational skills, and identity?
   c. What role did these perceptions have on how visitors engaged and interacted with the exhibits?
5. Control: In what ways and to what extent were visitors in charge of their own learning experiences?
6. Play: In what ways and to what extent were visitors able to be playful?

AUDIENCES

XI. Special Audiences

1. Did certain groups of visitors have different types of experiences, for example, those visitors with disabilities, traditionally underserved audiences, math novices or experts, math-phobes or math-lovers, different ages, etc.?
2. In what ways and to what extent were the exhibits more or less effective for different types of audiences?

[2] These motivational aspects are adapted from the *What Makes Learning Fun?* framework.

C. Museum Professional Experience

I. Team Member Thinking

1. What was it like being part of the *Math Moves!* collaboration? What worked and what didn't and why?
2. What did museum professionals learn about how visitors learn math?
3. How will that influence their future work with and for visitors?
4. In what ways and to what extent did museum staff who did not participate in the planning and development of the exhibits learn to look at math and math learning in new ways?
5. How did participation in MathCore contribute to one's own professional development?
6. How did partners come to recognize math in museum settings in new ways? What helped them to see math learning by visitors in new ways?
7. What new images of math were fostered as a result of MathCore? of ratio?
8. What role did body motion played in partners' developing understanding of mathematics and math learning?
9. How did the Math Core project affect each museum's staff's thinking about providing ISE experiences designed for repeat visitors?

II. Design/Development Process

1. What was the design/development process like, and how did this work/not work?
2. While developing their exhibits, to what extent was each partner aware of being part of the collaborative and how did that influence their work?
3. How did the Math Core partners see the interplay between experiential and content goals?
4. In what ways and to what extent did this influence the final exhibit designs?
5. How was designing and developing MathCore exhibits different than what they've done before?

III. Project Collaboration

1. What was the process used to ensure collaboratively developed exhibits and collaboratively planned evaluation?
2. How was the collaboration structured in ways that differed from more typical museum collaborations and exhibit development processes? Follow the history of the collaboration from grant-writing to production and beyond. What were the key elements leading to success?
3. How did different team members contribute differently?
4. How was it possible to both work collaboratively and still take account of each museum's individuality?
5. What were some key incidents/issues that had an impact on the collaboration?
6. How did the partners support each other? What tensions arose and how were they resolved?
7. What was the role of the non-museum partners and the advisors in this collaboration, and how did it differ from their usual roles?
8. How do the final exhibitions and evaluations reflect the collaborative process?
9. How could this model of collaboration be used in other projects and by other museums?
10. What would we do differently (both as individual museums and as members of a collaboration)?

D. DESIGN OF EXHIBITS

1. What was the design process? What strategies were used in the development of the exhibits and the feedback station, to ensure they were: open-ended, enduring, unlimited, unexpected, conversational, transactive, accessible, multi-sensory, kinesthetic, personally relevant, rewarding, surprisingly enjoyable, conducive to adult-child scaffolding?
2. What strategies were used to help visitors make connections between embodied understanding of mathematics and more abstract knowledge?
3. What worked and what didn't?
4. In what ways and to what extent did visitors perceive these exhibits similar to and/or different from other science museum exhibits?
5. In what ways and to what extent were the *Math Moves!* exhibit units: open-ended, enduring, unlimited, unexpected, conversational, transactive, accessible, multi-sensory, kinesthetic, personally relevant, rewarding, surprisingly enjoyable, conducive to adult-child scaffolding?
6. What design elements contributed to repeat visits?
7. In what ways and to what extent were the exhibit designs "rich" enough?
8. What aspects of the design of the exhibits contributed to meaningful conversation and teaching/learning engagements?
9. What aspects of the design of the exhibits contributed to the development of appropriate challenges, questions, and problems to solve?
10. Did some exhibit units foster repeat visits more or less than others? What contributed to this?

E. THE STUDY

1. What was it like being a recruited respondent in this study?
2. What worked well, and what didn't work as well?
3. What did we learn about planning and conducting a longitudinal evaluation study such as this?

F. LESSONS LEARNED

What have we learned about….

1. Designing and conducting a longitudinal and multi-site evaluation study?
2. Being part of a collaborative exhibit design/development process?
3. The feasibility and role of math exhibitions in science museums?
4. Working with external evaluators?
5. Using a qualitative naturalistic evaluation methodology?
6. Designing exhibits for repeat visits?

Appendix F: Descriptions of Recruited Respondents

One of the challenges of conducting a study such as this is that while as researchers we get to know each respondent quite well over the course of the project, it is easy for the study findings to appear detached from real human beings and their lives. Throughout the course of the study we were continually amazed and humbled at the level of commitment and dedication displayed by the participants. In an attempt to humanize all sixteen of the respondent families, we are including here a brief description of each family, along with photographs. These descriptions can't do justice to what the respondents so generously contributed, but we are committed to honoring how readily they shared their lives with us over this year and a half.

The core group was a mother and a daughter who was nine years old and in 4^{th} grade at the beginning of the study. A very active younger brother came along on some visits. Both the mother and daughter spoke English and Spanish; the daughter sometimes translated for her mom. The daughter was enrolled in an after-school science program at the museum. She liked and did well in math at school, where she was working with fractions and solving simple equations at the beginning of the study. The mother also liked math, although her formal education was limited. Both daughter and mother liked to draw; the daughter said one reason she liked math was because she could draw things like fractions. The daughter had been to *Math Moves!* with her after-school class prior to the initial study visit.

The focus child was a boy, six years old and a homeschooled Kindergartner at the beginning of the study. His mother accompanied him on all visits, and his father and younger sister also came along a couple of times. The focus child had attended homeschool science classes at the museum for several sessions prior to the initial study visit, but they had not seen *Math Moves!* prior to their initial study visit. His mom said he loves math and did really well in the subject in school. At the time of the final study visit he was doing addition and a little subtraction as part of his homeschooling, but they were not yet doing multiplication.

The focus child, seven years old and finishing up 1st grade at the beginning of the study, came with her mother on all visits. They were sometimes accompanied by the focus child's younger and older sisters, as well as her father and aunt. The family often spoke Spanish at home, but both mother and daughter spoke English most of the time in *Math Moves!* The focus child was in the bilingual class at school, and she learned math in Spanish. The daughter was somewhat neutral in her feelings about math, but the mom did not like it much at all. During the initial study visit the focus child showed some familiarity with the addition/subtraction, but was not yet doing multiplication.

The focus child had just turned nine years old at the time of the initial study visit and had just completed homeschooled 3rd grade. By the final study visit he was preparing to enter 5th grade at a regular school. He visited *Math Moves!* with his mom each time. His younger brothers also came along for a couple of visits, although they spent most those visits elsewhere in the museum. The focus child's homeschooling included participating in science programs and exhibits at the museum, and taking math classes at various summer camps and learning centers (including algebra and geometry). The focus child had visited *Math Moves!* a couple of times prior to the initial study visit. He seemed to enjoy experimenting with the exhibits, exploring the science and engineering of the exhibits as well as the math, and usually worked independently from his mom. By the final study visit he was applying multiplication and some division at the exhibits, but was unfamiliar with the concept of ratios, which they had not yet studied in school.

There were two focus children, twin boys who were 11 years old and in 5th grade at the beginning of the study, but on some visits only one boy could attend. They visited with their grandmother. Both twins liked and were good at math, as were their parents. The boys played chess and designed games, both in afterschool clubs and on their own or with their dad. They had been coming to the museum with their grandmother every other week since they were two years old. The grandmother was heavily involved with education and charter schools. She didn't have a personal overriding passion for math, but her work included budgeting and finances, and she had strong estimation skills. The boys were unfamiliar with ratios on their initial visits, but they knew about doubling and were good at multiplication.

The focus child was eight years old and in 2nd grade at the beginning of the study, and he visited *Math Moves!* with his father. The boy enjoyed math and loved playing with numbers—at home he even played with his school's computerized math program on his own. His father was a teacher who enjoyed language more than math. The family was a member of the museum and visited at least six times a year. The focus child could double and do simple multiplication at the beginning of the study, but he had not yet learned about ratios at school.

The focus child in this group was seven years old and finishing 1st grade at the beginning of the study. She visited with her mother, who also helped her with the small amount of math homework she brought home from school most nights. Mom felt challenged by some of the homework's word problems and logic questions, but even when they had trouble figuring things out, the focus child did not get frustrated or intimidated and kept her positive attitude towards math. The focus child enjoyed going places with her parents, and they came to the museum about 10 times a year.

The focus child in this group was a girl who was six-years-old and finishing Kindergarten at the beginning of the study. She visited *Math Moves!* with her grandmother, and her grandfather also came along on some visits. The girl liked math and was fairly advanced for her grade. She had been doing *JumpStart* math on her computer and, with some help, could work at the 2nd grade level. The grandmother, however, did not do well in math at school and felt math was a weak spot for her. Prior to the study the focus child had been coming to the museum about once a month with her grandmother.

The twin girls in this group were six years old and in Kindergarten at the time of the initial study visit. Their father, who said he loved math and was in a math-focused career, accompanied them on all visits. Their mom came along on two visits, and they got to show off the exhibition to a favorite aunt during one interim visit. The dad described the family as very involved with informal science activities, including visits to zoos and museums, watching *Nova*, and answering science questions using the Internet. The dad said he had visited the museum as a child, and he brought his family here at least three times a year. The girls however, differed in their interest in math and science, with one focusing more on reading and art than the other. However, both girls loved to measure things at home, where they had their own tape measure. By the final study visit the more math-focused twin was just beginning to do some division, which she had learned on her own.

The focus child in this group was 11 years old and had just completed 5th grade by the initial study visit. Her younger brother was an enthusiastic participant on all the study visits, and her younger sister came for the final study visit. Her parents, immigrants from Africa, spoke several African and European languages. Both parents came during the initial and final study visits, and they each accompanied the children on two of the interim visits. The daughter had been a little behind in math at school, but had been catching up and was eager to learn more. By the final study visit the focus child was doing well with basic multiplication and division at the exhibits; she had learned about ratios at school and recognized that they were part of *Math Moves!* The dad seemed to have both enthusiasm and a natural talent for mathematical thinking, which he enjoyed sharing with his children. The mom was more of a language person than a math person.

This daughter-mother dyad included the focus child, who was seven and had just completed 2nd grade by the time of the initial study visit, and her mom, who came on every visit. The focus child's older brother and her friend from school also came along on one interim visit. The daughter liked math and was in the high math group at school, but she sometimes got frustrated when she had trouble getting the answers. However, she enjoyed math when she was successful at solving problems. The mom had enjoyed school math but did not use it much at present. She had not taken courses beyond algebra By the final study visit the focus child was doing well using multiplication at the exhibits; although she was not doing much division at school, her mom found ways to explain the ratios at some exhibits using only multiplication.

The focus child was 10 years old and had just completed 4th grade by the initial study visit. His mom, who spoke English and Spanish, accompanied him on each visit, as did his younger sister. Two of his cousins came along on an interim visit. Although the focus child was about average in his math abilities—and had sometimes needed extra help—math was his favorite subject. At the time of the final study visit, he was doing multiplication and division at school, including fractions and a little bit of percentages but had not studied ratios yet. The mom had been good with math at school but did not use it much at present, at home or work.

The focus child in this group was 9 years old and in 5th grade at the start of this study. His mom, who like her son spoke both English and Spanish, accompanied him on the visits to *Math Moves!* The focus child's older sister and her friend also came along on some visits. His mom was a language teacher, and her son was studying Chinese in school. The focus child was very accomplished with computers, having skills with programs like PowerPoint and Skype, but his parents enforced limits on how much time he could spend using them at home. The family had seen *Math Moves!* once prior to the study, when it first opened.

[Image Consent Withheld]

The focus child in this group was a boy, 10 years old and in 5th grade when recruited for the study. His mother and usually his father accompanied him on *Math Moves!* visits. His younger brother was also an active participant in the study visits. The family was bilingual, speaking English at the museum and Farsi at home. The focus child liked math and was in math enrichment classes at school. He enjoyed his math homework, and his mother sometimes gave him additional math assignments to do at home. The mother was a health sciences researcher and had studied math intensively at university as part of her training. The family was a member of the museum.

This group included the focus child, seven years old and in 2nd grade when recruited for the study, and her mom, who came on every study visit. A younger sister also came along on *Math Moves!* visits. The focus child enjoyed school and was at grade level at math and a bit behind in reading. This family participated in museum programs and came to the museum a couple of times a year.

The core group included the focus child, 9 years old and in 4th grade when recruited for the study, and her mother. Some additional family/friends accompanied them on a couple of visits. The focus child did pretty well with math at school, but was more verbally and artistically inclined. The family had been museum members in the past and often visited on museum free days.

Appendix G: Host Venue Instructions

This study would not have been possible without the commitment of the four host institutions. Each museum had a liaison who worked closely with the evaluators to ensure things went smoothly for all recruited respondents. The *Host Venue Instructions* document below outlines the responsibilities undertaken by each of the four museum liaisons.

Host Venue Instructions

2/17/12

Your job:

- Develop a recruitment plan including text for email and a flyer.
- Document the exhibition.
- Recruit respondents.
- Work with Deborah to schedule spring site visits.
- Participate in periodical meetings.

Documentation to be gathered by each host venue:

- a list of all exhibit units in the Math Moves! exhibition
- photographs of the installations
- any significant changes to the exhibition that take place over the next two years, e.g. re-location of an exhibit or re-configuration of the floor plan, permanent removal or addition of an exhibit unit, etc.
- when an exhibit unit is off the floor for maintenance or otherwise unavailable for public use
- an inventory of all special programs and events
- any special materials created or added
- floor plans
- all memos, emails, communications from visitors, including comments posted on the feedback stations
- promotional materials distributed to the public

Recruitment/Selection/Contracting Process: We discussed the process that will be used for recruiting, selecting, and contracting with the RRs. We agreed that it will look something like the following:

1. Museum reps "advertise" the study to potential Recruited Respondents.
2. Potential respondents get in touch with museum reps. Museum reps have initial conversations with potential RRs to answer questions, gather information, and see if they fit the minimum criteria. For the child:
    - name
    - gender
    - fluency in languages other than English
    - anything that might prevent them from participating
    - special needs
    - experiences with and attitudes toward mathematics. Note that we are looking for participants with a wide range of experiences with and attitudes towards math. A nonjudgmental question might be: "We are interested in people with all kinds of math experiences. What is your child's math experience?"

   For the adult:
    - all of the above, plus
    - email address
    - phone
    - commitment to complete 6 study visits with focus child including follow-up activities
    - availability to meet during first site visit
3. Each museum rep brings a list of 5 potential RRs to the CET for consideration.
4. CET reviews and discusses the 20 potential RRs and selects 8-20 families to be officially invited to be RRs.
5. Museum reps contact the invited RRs (a) to determine if they are still interested, and (b) to discuss the terms for participating.
6. Museum reps make sure each RR understands the RR Contract & Consent Form.
7. Museum Reps get the appropriate signatures on all consent forms. This can take place either during a face to face meeting at the museum, or via mail.
8. Museum reps schedule the first study visits to coincide with the first SRA site visit.

Appendix H: Sample Recruitment Flyer

The following pages include the flyer that was used to recruit respondents at the Museum of Life and Science in Durham. Each institution developed their own flyer but they all shared similar narrative that was adapted to each particular museum.

Plan for Recruited Visitors — Museum of Life and Science

Recruitment
We will target two audiences: current members and our 'Ignite Learning' member families[1].

Current members: We will include a brief announcement in our member e-newsletter:

> The Museum of Life and Science is working with partner museums across the nation to investigate how families interact with math exhibits over time. We are looking for a few families **with children between 6 and 11 years of age** to visit the museum, use our math exhibits, and talk with researchers. Each family will be asked to make several scheduled visits over two years. Families will receive 1) a free Explorer 4 Membership for each year of the study, 2) vouchers for meals at Sprout Café for each visit during the study, and 3) an additional one-year Explorer 4 Membership after the study's completion.

We also will work with our membership director (Janet Hoshour) to email a recruitment letter to member families with at least one child between 6 and 11 years of age. The recruitment letter will describe the project and the expectations of and compensation for participating families. ***A sample recruitment letter is attached.***

Ignite Learning Partners: The Museum's Ignite Learning program serves historically underserved communities in Durham. We will email a request to several Ignite Learning partner organizations informing them about the research study and asking them to distribute recruitment letters to their constituents. The letter to Ignite Learning partner organizations follows:

> The Museum of Life and Science is working with partner museums across the nation in an NSF-funded project to investigate how families interact with math exhibits over time. We are looking for a few families **with children between 6 and 11 years of age** to visit the museum, use our math exhibits, and talk with researchers. Each family will be asked to make several scheduled visits over the two years. Families will receive 1) a free Explorer 6 Membership for each year of the study, 2) vouchers for meals at Sprout Café for each visit during the study, 3) mileage and/or public transport reimbursement for travel expenses to and from the Museum for each visit during the study, and 4) an additional one-year Explorer 6 Membership after the study's completion.
>
> I've attached a letter with additional information about the study. Please share this information with anyone at your organization who might be interested. A limited number of families will be accepted into the study. As noted in the letter, interested families should contact Elizabeth Fleming at elizabeth.fleming@ncmls.org.
>
> Please let me know if you have any questions. I hope some of your families will take advantage of this opportunity.

We will include a recruitment letter similar to the one we use for current members. The only difference is that Ignite Learning member families will be offered an Explorer 6 Membership in order to be consistent with the current Ignite membership offering, and reimbursement for travel expenses to and from the Museum.

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[1] This membership initiative serves historically underserved communities in Durham.

Selection
Families interested in learning more about the study are asked to contact Elizabeth Fleming, Exhibit Development Manager at MLS. She will review with an adult member of each family the expectations of and compensation for participants and answer any questions they may have. If the family is interested in participating in the study, she will collect information about the family.

- Information about the *focus child* will include: name, age, gender, fluency in languages other than English, anything that might prevent him or her from participating, special needs, and the child's experiences with and attitudes toward mathematics. (Note that we are looking for participants with a range of experiences with and attitudes toward math. A nonjudgmental question might be: "We are interested in people with all kinds of math experiences, what is your child's math experience?")
- Information about the *accompanying adult* will include: name, email address, phone number, their commitment to complete six study visits with the focus child including follow-up activities, fluency in languages other than English, anything that might prevent him or her from participating, special needs, availability to meet during the first site visit, and his or her experiences with and attitudes toward mathematics. (Note that we are looking for participants with a range of experiences with and attitudes toward math. A nonjudgmental question might be: "We are interested in people with all kinds of math experiences, what is your math experience?")

The Museum of Life and Science will collect information about five families for possible participation in the study. Using the information, Deborah Perry, the study's lead investigator, and the rest of the Core Evaluation Team will select 2-5 families for participation.

Getting Started
Each of the families selected for participation will be asked to meet with Fleming and Perry during Perry's first site visit. Prior to this meeting, Fleming will arrange for each of the selected families to receive a recruitment package which will contain an informed consent form, consent/release forms for audio and video data collection, and an agreement that provides details of the participants' expectations and compensation. Fleming will review the documents with the parent or guardian of the focus child and obtain his or her signatures. This may be done in person or by mailing the recruitment package to the family and discussing it by phone. No data collection will begin until a parent or guardian of the focus child signs the agreement and consent forms.

Study Visits
Although families will be able to visit the museum for free at any time (see Compensation section, below), they will be asked to participate in at least three *study visits* each year of data collection. Prior to each study visit, a member of the family will contact Fleming to let her know when they will be visiting the museum. This will allow Fleming to prepare café vouchers that the family will receive during the visit (see Compensation section) and for

Perry to arrange post-visit procedures, such as scheduling a phone interview or providing a survey or activity for the family to complete. Each study visit to the museum must include the focus child and the accompanying adult.

Compensation

For each of the two years of the study, each family will receive a membership to the museum. An Explorer 4 membership covers admission of up to 4 people (children under age 3 are always free, and are not included in the membership count).[2] Also, for each study visit, families will receive vouchers for meals at Sprout Café. The café vouchers are valid only on the day of the study visit.

The annual membership will be given to the family at the first meeting and renewed after the first year of participation in the study. Families who complete the full two years of the study will receive a family membership (but no café vouchers) for one additional year. Families who leave the study during the first year will keep their membership for the remainder of the year, but will not receive any additional café vouchers and will not have their complimentary membership renewed; families who leave the study in the second year will not receive any additional café vouchers and will not receive a complimentary third year of membership.

[2] Membership includes: free admission to the Museum and *Magic Wings* Butterfly House (excluding group admission); discounts and advance registration to educational programs, including Summer Camp; invitations to special members-only events and exhibit previews; 10% Discount in Museum Gift Shops; Birthday Party discounts; Subscription to Museum e-newsletter; free or reduced admission to more than 280 science museums worldwide (does not include those within 90 miles - see ASTC Passport Program; members-only early admission 9:00-10:00 a.m. Monday through Saturday (except closed Mondays September through December); unlimited and behind the scenes access to the Science Education Resource Center. (Note: Memberships are not transferable. At least one Member named on the membership card must be present and included in the admission count when visiting the Museum. Up to two adults living in the same household may be listed on the membership card.)

We Are Looking For Families To Participate In An Exciting Research Project

March 5, 2012

You and your family may be interested in an exciting research study about math exhibits in science centers, a project called Math Core for Museums funded by the National Science Foundation. Researchers are investigating how youngsters and families interact with math exhibits over time. The project involves four science centers across the United States that developed cutting-edge *Math Moves!* exhibits, which just opened at the Museum of Life and Science. The research team needs *your* help to understand how children and their families learn from them.

If you have at least one child between 6 and 11 years of age, you may be eligible. Only a limited number of families will be selected to participate. Here's the scoop:

Q: What is my family being asked to do?
A: We would like to observe the members of your family as they interact with our new *Math Moves!* exhibits and then ask you some questions about your experience using them. Some interviews will be conducted at the museum immediately after using the exhibit, and some will be conducted soon after by phone or by completing a written survey or activity. Also, we may want to videotape your family using the exhibit or during the interviews. (All information will be strictly confidential.) Each interview is expected to take about one hour. **We are interested in families who will agree to visit the math exhibits and talk with a researcher 6 times over the next two years.**

Q: Do we have to pay admission and parking for each visit?
A: No. We will provide you with a family membership during your participation in the study. The Explorer 4 Membership provides your family (up to 4 people over the age of 3, under age 3 are always free) with free entry to the Museum any time during the two years of the study. During each of your family's 6 study visits, we'll also provide you with vouchers to purchase lunch or snacks for your family at Sprout Café.

Q: Can my whole family come?
A: Participant groups can be as small as 2 members and as large as 4 members. One member of the family should be between 6 and 11 years of age and will be expected to participate in all 6 of your study visits to the museum. This person will be the "focus child." Also, one adult member of your family should participate in all of the study visits. This person will be the "accompanying adult." Your family visits may include other children and adults.

Q: What are the language requirements?
A: Either the child or the adult who participates in all of the museum visits should be proficient in English.

Q: Why might my family want to participate?
A: First of all, we think the exhibits are fun, engaging, and enriching ways to learn about key math concepts. Second, we will give you a free family membership for both years of the study. The family membership provides free entry to the Museum at any time for up to 4 people. Plus, we'll renew your membership for another year after completing the study. Finally, you and your family will be making a contribution to nation-wide efforts to improve opportunities for children and their families to learn math. All participants' names will be confidential and we will not identify anyone in reports on the study. We'll be pleased to send you a report of our findings when the study is completed.

Q: Okay, we're interested! Who should we contact?
A: For more information, please email Elizabeth Fleming at the Museum of Life and Science: elizabeth.fleming@ncmls.org.

Appendix I: Recruitment Information Form

After collecting the names and contact information of potential respondents for the study, each of the four museums selected a small number to talk with on the phone and assess whether or not they would be good candidates for the study. The following pages include information and questions to guide and assist museum staff in this initial selection.

Math Moves! Longitudinal Study
Recruitment Information Form
3/5/12

The following information will be helpful as the Core Evaluation Team selects respondents to participate as Recruited Respondents. The more information we have about each of the candidates, the better able we will be to choose families that will help us understand the exhibits and how they work for different types of visitors.

The list of questions below do not need to be asked in exactly this order or using these exact words. We envision your communication with potential respondents to be more like a friendly conversation. We do need all this information, but feel free to adapt the questions so that they are comfortable and flow easily.

When talking with candidates, please make sure they understand that only a few folks will be selected to be part of this study. Although they might be perfect candidates, they still might not be selected. They also need to understand that this will take a significant chunk of time, and that the focus child and the accompanying adult need to be the same people throughout the study. If they aren't absolutely sure they will be able to commit for the full two years (for example, there's a slight chance they will be moving before the end of 2013), still take down their information and note their concerns. If however, they aren't sure they can (for example) make repeat visits to the museum, or engage in the follow-up interviews and activities, then they probably are not appropriate candidates. Finally, they need to understand that over the two years they will be working with someone from Selinda, not with you; although they also need to be reassured that you will remain their link with the museum, and that you will be in close touch throughout the process.

Here are the general categories of information we are looking for:
a. Basic demographic information: name, age, gender, grade, school, location, languages, etc.
b. Relationship with math: attitudes, understandings, experiences
c. Relationship with the Museum: visitation pattern, attitudes, experiences
d. Any special interests/challenges: What makes this family group unique?
e. Contact Information: email, snail mail, phone, best days/times
f. Other: anything else you think will help us select the best respondents

Date: *Person gathering information*: *Institution*:

First, about your child:

1. What is your child's name? Gender? F M Other
2. What is your child's age?
3. What grade are they in?
4. What kind of school does your child go to?
5. What languages is your child fluent in?
6. What's your child's experience with math and/or math-related activities? Think also of experiences outside of school. (We are looking for people with all kinds of math experiences.)
7. What's your child's attitude towards math? (We are looking for people with all kinds of math experiences and attitudes.)
8. Has your child ever been part of a study like this before?
9. Does your child have any special needs?
10. What is your relationship to your child? (Mother? Father? Aunt? Sibling? Guardian? Etc.)
11. If you are not the legal parent/guardian, who do we need to contact to get consent?
12. Is there anything you can think of that might prevent your child from participating?
13. What else should we know about your child and their interest in participating in this study?

Now, about you.

1. What is your name? Gender? F M Other
2. What languages are you fluent in?
3. What is your experience with and attitudes towards math? Again, we are looking for people with all kinds of math experiences and attitudes.
4. How often have you been to the museum? Are you a member? Again, we are looking for all kinds of people.
5. Are you able to bring your child to the museum at least six times, three times each year for the next two years?
6. Do you have any special needs?

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7. Is there anything you can think of that might prevent you from participating?
8. What questions do you have?
9. Is there anything else you think might be important for us to know?
10. Contact Information:
    a. Email
    b. Phone
    c. Snail mail
    d. Which of the above do you prefer we use when we need to be in touch?
    e. Best days/times to be in touch?

For Internal Use Only:

Now that you're done talking with the potential respondent, in your opinion:

1. How willing and able will this family be to engage in meaningful conversations about their experiences with the MM exhibits?
2. What did you find most interesting about this family group?
3. What do you think would be the best thing about working with this family group?
4. What do you think will be the most challenging?
5. What else should we know about this family group? The more information we have, the better selection we can make.
6. Do you think they will or will not make a good respondent for this study? Why or why not?

Appendix J: Sample Consent & Agreement Form

Each recruited respondent group received, reviewed, and signed a Consent & Agreement Form such as the one included here. Each form was identical, except for the museum's logo and museum rep name and contact information. Tokens of appreciation also varied by museum (item #9 on page 3).

***Math Moves!* Evaluation and Research Study**

Parental Informed Consent Form

You and your child are invited to participate in an evaluation/research study being conducted at four museums across the country, including the Science Museum of Minnesota. The following information is provided to help you make an informed decision about whether or not to participate. If you have any questions, please do not hesitate to ask.

You and your child are eligible to participate because (a) you are the parent or legal guardian of a child, (b) your child is between the ages of 6 and 11, and (c) you are able to visit the museum with your child at least 3 times this year (2012) and 3 times next year (2013). You and your child have been selected because we believe you may be able to give us useful insights into how the *Math Moves!* exhibits help young people and their families use and understand math over time.

During each of your study visits to the *Math Moves!* exhibits you will spend as much time as you want in the exhibition using whichever exhibits you want, in whatever way you choose. After each of your study visits to *Math Moves!*, you and your child will complete a brief written activity or survey about your visit, or you will talk about your visit with a researcher, either in person or by telephone. Sometimes you may be asked to both write something and talk about your visit. Sometimes we may also observe you and your child using the exhibits, but you will always be told if we are going to observe you. Additional details are explained in the attached Agreement.

There are no known risks associated with this research. Any information obtained during this study that could identify you or your child will be kept strictly confidential, unless you specifically request that such information be released. The information obtained in this study may be published in museum and/or education journals, or presented at museum and/or education meetings, but you and your child's identity will be kept strictly confidential.

You and your child are free to decide not to participate in this study, or to withdraw at any time with no adverse consequences.

You and your child have voluntarily made a decision whether or not to participate in this evaluation/research study. Your signature certifies that you and your child have decided to participate, having read and understood the information presented. You will be given a copy of this consent form and the attached documents to keep.

Math Moves! Recruited Respondent Contract & Agreement

____ ___________________________

Signature of Parent/Guardian Date

In my judgment the parent/legal guardian is voluntarily and knowingly giving informed consent and possesses the legal capacity to give informed consent to participate in this evaluation/research study.

____ ______________________

Signature of Investigator Date

Attachments:

- Agreement
- Adult Participant Release Form
- Child Participant Release Form

Math Moves! Evaluation and Research Study

Agreement

1. I agree that my child and I will participate in this evaluation/research study.
2. For the purposes of this study I will be considered the *accompanying adult* and my child will be considered the *focus child*.
3. My child and I will visit the *Math Moves!* exhibition at least 3 times each year in 2012 and 2013, for a total of at least 6 *study visits*. (A *study visit* will consist of any visit to the *Math Moves!* exhibition by the focus child and the accompanying adult where they engage with any one or more of the *Math Moves!* exhibits for a reasonable amount of time and complete after-visit activities.) Other family members and friends may join the accompanying adult and focus child.
4. Prior to each study visit, I will get in touch by phone or email with Steven Guberman at the Science Museum of Minnesota (at least 48 hours in advance if possible) so that he can arrange for our visit. I understand that my child and I may do more than 3 study visits each year, but each time we do, I will contact Steven. I also understand that my child and I may visit the museum at any time not as part of a study visit and, in those situations, I do not need to let anyone know.
5. During some study visits my child and I may be observed while we are using the *Math Moves!* exhibits.
6. After each study visit my child and I will either complete a written activity (expected to take no more than 30 additional minutes), or talk with a researcher, either face-to-face or on the telephone (expected to take no more than 1 hour). After some study visits I will be asked to complete a written activity *and* talk with a researcher.
7. As part of this study we will be asked for permission to audio and/or video record the interviews. I understand that there is a possibility that not all interviews will be audio- or video-recorded. I also understand that I may refuse to be recorded, and that my refusal will not jeopardize or otherwise compromise our participation in the study in any way.
8. All information obtained about me and/or my child as part of this study will be kept strictly confidential. Although the information may be used as part of research reports, publications, and/or presentations, my child and I will not be identified. (See the attached Adult and Child Release forms.)
9. I understand that for our participation in this study I will receive the following:
    a. A free **household membership** for both years of the study and, upon completion of the study, an additional year of free membership, for a total of three years of membership if we remain in the study for the entire two years. I understand that if my child and I decide to withdraw from the study, we will keep our free membership for the remainder of that year, but we will not receive any additional year(s) of free membership. I understand that the household membership covers two adults and any children or grandchildren in the family under 18 years of age, and provides for admission to the museum and tickets to the Omnitheater, and that it may be used whether or not we are visiting the *Math Moves!* exhibits. The museum's membership policies are available at www.smm.org/members/policies.
    b. Vouchers to purchase **lunch or snacks** for my family in the museum's cafeteria during each of our study visits. For each study visit, the museum will provide up to five vouchers, each voucher is good for purchases up to $12. The vouchers can be used at any

of the museum's dining facilities (Elements, Chomp, and Java Lab) only on the day of the study visit.

c. A voucher to use in the museum's **parking** structure during each study visit, or alternatively, reimbursement for the cost of **transportation** to and from the museum.

d. A copy of the **final evaluation/research report** if I request it.

10. I understand that additional family members may participate in a study visit.
11. I understand that when my family and I visit the museum not as part of a study visit, my household membership will still be valid, but I will not receive vouchers for food, parking, or transportation.
12. I understand that if I have any questions at any time I can get in touch with Dr. Steven Guberman at the Science Museum of Minnesota:
    Phone Number: (651) 221-4728
    Email address: sguberman@smm.org

Steven R. Guberman
Senior Evaluation and Research Associate
(651) 221-4728
sguberman@smm.org

Adult Participant Release Form

The study you are being asked to join is being conducted by Selinda Research Associates with the Science Museum of Minnesota (SMM). To help SMM learn about the effectiveness of their math exhibits, researchers will be collecting video recordings and/or audio recordings for research and evaluation purposes. The collected videos and audios will be used as data for research and evaluation studies that investigate how families use math exhibits over time. The video and audio recordings (and transcriptions from them) may be published in research journals or conference presentations as a part of this research. Your name will never be associated with your image and/or comments unless you request that we do so. You will not receive any monetary compensation for the use of your image or voice recordings.

If you do not wish to be video or audio recorded, please indicate so on the attached form. Your participation in this research study is voluntary and you can participate in the *Math Moves!* exhibits without being part of the study.

You have the right to withdraw consent from this study at any time during or after your participation. Your audio and video recordings will not be used for one week or more following your participation so that you have time to reconsider your participation before your image or voice is made public. If you have any concerns or questions about the study, you may contact Dr. Steven Guberman, Senior Associate in the Department of Evaluation and Research on Learning at the Science Museum of Minnesota, using the contact information above.

1. Consent for Videography (check one)

❑ *Yes, I* ***agree*** *to be video recorded and to have the videos used for research, publications, or other purposes as detailed in the release form above. I understand that I will not receive monetary compensation for the use of these images.*

❑ *I* ***do not*** *agree to be video recorded during this research study.*

2. Consent for Audio Recordings (check one)

❑ *Yes, I* ***agree*** *to be audio recorded and to have the audio recordings used for research, publications, or other purposes as detailed in the release form above. I understand that I will not receive monetary compensation for the use of these audio recordings.*

❑ *I* ***do not*** *agree to be audio recorded during this research study.*

Printed Name: ____

Signature: ____ Date: _____________________

Email Address or Phone Number:____

Steven R. Guberman
Senior Evaluation and Research Associate
(651) 221-4728
sguberman@smm.org

Child Participant Release Form

The study your child is being asked to join is being conducted by Selinda Research Associates with the Science Museum of Minnesota (SMM). To help SMM learn more about the effectiveness of their math exhibits, researchers will be collecting surveys, interviews, video recordings, and audio recordings for research and evaluation purposes. The collected surveys, interviews, videos, and audios will be used as data for research and evaluation studies that investigate how families use math exhibits over time. The surveys, interviews, video recordings, and audio recordings (and transcriptions from them) may be published in research journals or conference presentations as a part of this research. Your child's name will never be associated with his or her image or comments unless you and your child request that we do so. You and your child will not receive monetary compensation for the use of your child's survey, interview, image, or voice recordings.

If you do not want your child to be surveyed, interviewed, videotaped or audiotaped, please indicate this on the attached form. Your child's participation in this research study is voluntary and he or she can participate in the *Math Moves!* exhibits without being part of the study.

You and your child have the right to withdraw consent from this study at any time during or after the study. Your child's survey, interview, and audio and video data will not be used for one week following his or her participation so that you have time to reconsider your child's participation before his or her image/voice is made public. If you have any concerns or questions about the study, you may contact Dr. Steven Guberman, Senior Associate in the Department of Evaluation and Research on Learning at the Science Museum of Minnesota, using the contact information above.

1. Consent for Survey (check one)

❑ *Yes, I **agree** to allow my child to be surveyed and to have his or her surveys used for research, publication, or other purposes, as detailed in the release form above. I understand that neither I nor my child will receive monetary compensation for the use of these surveys.*

❑ *I **do not** agree to allow my child to be surveyed during this study.*

2. Consent for Interview (check one)

❑ *Yes, I **agree** to allow my child to be interviewed and to have his or her interviews used for research, publication, or other purposes, as detailed in the release form above. I understand that neither I nor my child will receive monetary compensation for the use of these interviews.*

❑ *I **do not** agree to allow my child to be interviewed during this study.*

3. Consent for Videography (check one)

❑ *Yes, I **agree** to allow my child to be video recorded and to have his or her video recordings used for research, publications, or other purposes as detailed in the release form above. I understand that neither I nor my child will receive monetary compensation for the use of these video recordings.*

❑ *I **do not** agree to allow my child to be video recorded during this study.*

4. Consent for Audio Recordings (check one)

❑ *Yes, I **agree** to allow my child to be audio recorded and to have the audio recordings used for research, publications, or other purposes as detailed in the release form above. I understand that neither I nor my child will receive monetary compensation for the use of these audio recordings.*

❑ *I **do not** agree to allow my child to be audio recorded during this study.*

Your Child's Name (Please Print): ____

Parent or Guardian's Signature: ______________________________ Date: ____________

Email Address or Phone Number: ____

Appendix K: Tokens of Appreciation by Institution

Each institution offered slightly different tokens of appreciation to Recruited Respondents (RRs) in keeping with each institution's culture and operations. Following is a summary of what RRs received at each museum.

Explora, Albuquerque, NM

- A free family membership for both years of the study. I understand that if my child and I decide to withdraw from the study during the first year, we will keep our free membership for the remainder of that year, but we will not receive an additional year of free membership. I understand that the family membership covers two adults and up to four children less than l8 years of age in the some household, and that it may be used whether or not we are visiting the *Math Moves!* exhibits. The museum's membership policies are available at http://www.explora.us/en/mn-join-give-en/mn-membership-en
- A 50$ Explora gift certificate at the end of the two year project and upon completion of the 6 study visits.

Museum of Science, Boston

- Over the course of the study, each fully participating family will receive two Basic 5 memberships to the museum. A Basic 5 membership admits up to 5 people to the Exhibit Halls every visit, and includes 2 Mugar Omni Theater passes, 2 Planetarium passes, and 2 Exhibit Hall guest passes, and reduced fees for special exhibitions. Also, for each study visit, families will receive reimbursement for parking fees.
- The annual membership will be given to each successfully participating family after the first year of participation in the study. Families who complete the full two years of the study will receive a membership for an additional year. Families who leave the study during the first year will not receive a membership; families who leave the study in the second year will not receive a second year of complimentary membership.

Museum of Life & Science, Durham, NC

- For each of the two years of the study, each family will receive a membership to the museum. An Explorer 4 membership covers admission of up to 4 people (children under age 3 are always free, and are not included in the membership count). Also, for each study visit, families will receive vouchers for meals at Sprout Café. The café vouchers are valid only on the day of the study visit.
- The annual membership will be given to the family at the first meeting and renewed after the first year of participation in the study. Families who complete the full two years of the study will receive a family membership (but no café vouchers) for one additional year. Families who leave the study during the first year will keep their membership for the remainder of the year, but will not receive any additional café vouchers and will not have their complimentary membership renewed; families who leave the study in the second year will not receive any additional café vouchers and will not receive a complimentary third year of membership.

Science Museum of Minnesota, St. Paul:

- A free household membership for both years of the study and, upon completion of the study, an additional year of free membership, for a total of three years of membership if we remain in the study for the entire two years. I understand that if my child and I decide to withdraw from the study, we will keep our free membership for the remainder of that year, but we will not receive any additional year(s) of free membership. I understand that the household membership covers two adults and any children or grandchildren in the family less than 18 years of age, and provides for admission to the museum and tickets to the Omnitheater, and that it may be used whether or not we are visiting the *Math Moves!* exhibits. The museum's membership policies are available at www.smm.org/members/policies.
- Vouchers to purchase lunch or snacks for my family in the museum's cafeteria during each of our study visits. For each study visit, the museum will provide up to five vouchers; each voucher is good for purchases up to $10. The vouchers can be used at any of the museum's dining facilities (Elements, Chomp, and Java Lab) only on the day of the study visit.
- A voucher to use in the museum's parking structure during each study visit, or alternatively, reimbursement for the cost of bus or train fare to and from the museum.

ACKNOWLEDGEMENTS

This study would not have been possible without the passionate dedication and commitment of many individuals, organizations—Science Museum of Minnesota, Museum of Science (Boston), Explora, Museum of Life and Science (Durham, NC), San Diego State University, and TERC—and the financial support of the National Science Foundation. To all we are deeply grateful.

Museum staff and project advisors worked tirelessly to contribute to the evaluation in ways not typical of most museum evaluations. Staff and advisors participated in lengthy evaluation planning meetings, reviewed and revised documents, discussed and debated many of the more difficult concepts and issues (helping us advance our own fluency), developed recruitment flyers, recruited and screened potential respondents, arranged for respondent study visits, took care of the respondents during those visits including translating and assisting with the follow-up interviews when necessary, served as videographers and AV technicians, assisted during our on-site visits, and in some cases served as respondents themselves. In all the many museum evaluations Selinda has completed, never before have we seen such dedication, responsiveness, and true collaboration.

We know this list is incomplete but in addition to the entire *Math Core* team, we would like to thank in particular: Betsy Adamson, Armelle Casau, Sarah Cohn, Elizabeth Fleming, Steven Guberman, Heidi Heidgerken, Stephanie Iacovelli, Josh Larson, Anna Lindgren-Streicher, Troy Livingston, Roger McNew, Ricardo Nemirovsky, J. Newlin, Al Onkka, Gayra Ostgaard, Paul Tatter, Scott Van Cleave, and Tracey Wright. Our apologies to those people whose names we have neglected to include. It's not that your efforts went unnoticed or were unappreciated; only that our memories and notes proved insufficient. We extend a special thank you to all the staff respondents who cannot be named but gave so generously of your time and thoughts. You know who you are.

And of course our deepest gratitude goes to the sixteen family groups who served as recruited respondents for this study. We cannot name you by name, but we thoroughly enjoyed getting to know each and every one of you, and are humbled by your commitment to the project and deeply grateful for your unfailing generosity over many, many months. It is not hyperbole to say that without your contributions, this study would not have been possible.